NOTES: SECONDARY 2 HONORS

COMPLEX NUMBERS (5.2A/AH/B)

STARTER

1. Factor the polynomial completely. $x^4 - y^4$ $(x^2)^2 - (y^2)^2$ $(x^2+y^2)(x^2-y^2)$ $= \boxed{(x^2+y^2)(x-y)(x+y)}$	2. Is the polynomial quadratic in nature? Explain how you know. $3\sqrt[3]{x^2} - \sqrt[3]{x} - 2$ $= 3x^{2/3} - x^{1/3} - 2$ $u = \sqrt[3]{x}$ $3u^2 - u - 2$ yes
3. Given $f(x) = 3x + 3$ and $g(x) = x^2 - 5x - 6$. Find $\frac{f(x)}{g(x)}$ and state the domain. $\frac{f(x)}{g(x)} = \frac{3x+3}{x^2-5x-6} = \frac{3\cancel{(x+1)}}{\boxed{\cancel{(x+1)}}(x-6)} = \boxed{\frac{3}{x-6}}$ $x-6 \neq 0$ $x+1 \neq 0$ $x \neq 6$ $x \neq -1$ $\boxed{D: (-\infty,-1)\cup(-1,6)\cup(6,\infty)}$	4. Solve for x. $\frac{\cancel{2}(x+4)^2}{\cancel{2}} = \frac{6}{2}$ $\sqrt{(x+4)^2} = \sqrt{3}$ $x+4 = \pm\sqrt{3}$ $\boxed{x = -4 \pm \sqrt{3}}$

Vocabulary

- The **imaginary unit,** $i = \sqrt{-1}$
- The number system can be extended to include the set of complex numbers.
  A complex number written in standard form is a number $a + bi$, where *a* and *b* are real numbers.
- The '*a*' is called the real part of the complex number and '*b*' is called the imaginary part of the complex number.
- If a=0 (there is only an imaginary part) then the complex number is also called imaginary and if b=0 (there is only a real part) then the complex number is also part of the real numbers.

Example: Identify the sets of numbers to which each number belongs to

1. $\frac{2}{5}$
   real: $\frac{2}{5}$
   imaginary: $0i$
2. $i\sqrt{7} = 0 + i\sqrt{7}$
   real: 0
   imaginary: $i\sqrt{7}$
3. $\pi - \sqrt{2}i$
   real: π
   imaginary: $-i\sqrt{2}$
4. $\frac{7}{8} + \frac{4}{5}i$
   real: $\frac{7}{8}$
   imaginary: $\frac{4}{5}i$

Example: Simplify the radical by using the complex number system

1. $\sqrt{-9} = \sqrt{-1} \cdot \sqrt{9}$
   $= \boxed{3i}$
2. $\sqrt{-24} = \sqrt{-1} \cdot \sqrt{24}$
   24: 2, 12; 12: 3, 4; 4: 2, 2
   $= \boxed{2i\sqrt{6}}$
3. $-\sqrt{-72} = \sqrt{-1} \cdot \sqrt{72}$
   72: 36, 2; 36: 6, 6
   $= \boxed{-6i\sqrt{2}}$

Adding, Subtracting and Multiplying with Complex Numbers

Example: Addition and Subtraction

1. $(3+2i)+(5-4i)$

$= \boxed{8-2i}$

2. $(7-5i)-(-2+6i)$

$= \boxed{9-11i}$

Example: Multiplication

1. $-3(-7+6i)$

$= \boxed{21-18i}$

2. $(-2+9i)(-3-10i)$ FOIL

$= 6+20i-27i-90i^2$

$= 6-7i-90(-1)$

$= \boxed{96-7i}$

$i=\sqrt{-1}$

$i^2=(\sqrt{-1})^2=-1$

Finding Powers of "*i*"

i	i
i^2	-1
i^3	$-i$
i^4	1
i^5	i
i^6	-1

What pattern do you notice with the table on the left?

Example: Find the following:

1. $i^{293} = \boxed{i}$

$\frac{293}{4}=73.25$

R:1

$293 \div 4 = 73$, -28, 13, -12, 1

2. $i^{15} \boxed{=-i}$

$\frac{15}{4}=$ R=3

3. $i^{66} = \boxed{-1}$

$\frac{66}{4}=16.5=16\frac{2}{4}$

R:2

Vocabulary

- The CONJUGATE of a complex number is a number in standard complex form *a*+*bi*, where the imaginary part, *bi*, has the opposite sign of the original.

Example: Find the conjugate of the following complex numbers.

1. $4-2i$

$4+2i$

2. $7+8i$

$7-8i$

3. $7i$

$-7i$

4. 28

28

Example: Find the product of the complex number and its conjugate.

1. $6-2i$ conjugate: $6+2i$

$(6-2i)(6+2i)$

$=36+\cancel{12i}-\cancel{12i}-4i^2$

$= 36-4(-1) = \boxed{40}$

2. $-3i$ conjugate: $3i$

$(-3i)(3i)$

$= -9i^2 = -9(-1)$

$= \boxed{9}$

3. $-8+6i$ conjugate: $-8-6i$

$(-8+6i)(-8-6i)$

$= 64+\cancel{48i}-\cancel{48i}-36i^2$

$= 64-36(-1)$

$= 64+36 = \boxed{100}$

4. $-2-i$

$(-2-i)(-2+i)$

$=4-2i+2i-i^2$

$= 4-(-1)$

$= \boxed{5}$

5. $4-5i$

$(4-5i)(4+5i)$

$= 16+\cancel{20i}-\cancel{20i}-25i^2$

$16-25(-1)$

$= \boxed{41}$

6. $-1+9i$

$(-1+9i)(-1-9i)$

$= 1+9i-9i-81i$

$= 1-81(-1)$

$= \boxed{82}$