

GRAPHING COMPLEX AND DIVIDING COMPLEX NUMBERS (5.2BH/5.2CH)

VOCABULARY:

- **Modulus:** represents the distance a complex number is from the origin.

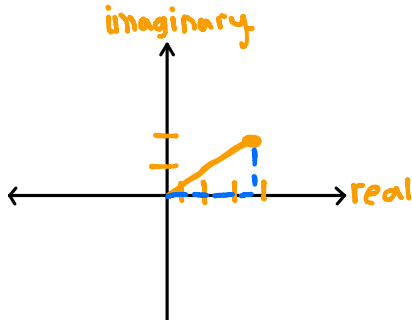
- The modulus of the complex number $a + bi$ is symbolized as $|a+bi|$

Example: Graph the following complex numbers on the complex plane and then find the modulus.

1. $4 + 2i$

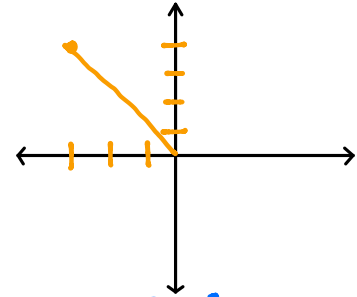
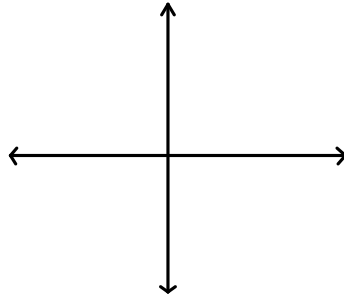
2. $2 - i$

3. $-3 + 4i$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4^2 + 2^2 &= c^2 \\ 16 + 4 &= c^2 \\ 20 &= c^2 \\ c &= 2\sqrt{5} \end{aligned}$$

$$|4 + 2i| = 2\sqrt{5}$$



$$\begin{aligned} (3)^2 + (4)^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ c &= 5 \end{aligned}$$

$$|-3 + 4i| = 5$$

From the example above, we see that $|a + bi| = \sqrt{a^2 + b^2}$

Example: Find the modulus of the following examples.

$$\begin{aligned} 1. |3 - 2i| &= \sqrt{3^2 + (-2)^2} \\ &= \sqrt{9 + 4} \\ &= \sqrt{13} \end{aligned}$$

$$2. |-6 + 4i|$$

$$\begin{aligned} 3. |-5i| &= \sqrt{0^2 + (-5)^2} \\ &= \sqrt{0 + 25} = \sqrt{25} \\ &= 5 \end{aligned}$$



DIVIDING COMPLEX NUMBERS

We cannot have an imaginary number in the denominator. So we need to multiply the denominator by the **conjugate** to get a real number. But remember, whatever you multiply the bottom by, you need to multiply the top as well.

Example: Divide each complex rational expression and write the answer in standard form.

$$\begin{aligned} 1. \frac{2}{8i} \cdot \frac{-8i}{-8i} &= \frac{-16i}{-64i^2} = \frac{-16i}{64} \\ &= \frac{-i}{4} \end{aligned}$$

$$2. \frac{6 + 8i}{9i}$$

$$\begin{aligned} 3. \frac{10}{(2+i)(2-i)} &= \frac{20 - 10i}{4 - i^2} \\ &= \frac{20 - 10i}{4 + 1} = \frac{20 - 10i}{5} \\ &= 4 - 2i \end{aligned}$$

$$\begin{aligned} 4. \frac{(22 - 7i)(4 + 5i)}{4 - 5i} &= \frac{88 + 110i - 28i - 35i^2}{(4)^2 - (5i)^2} \\ &= \frac{88 + 82i + 35}{16 + 25} = \frac{123 + 82i}{41} \\ &= 3 + 2i \end{aligned}$$

$$\begin{aligned} 5. \frac{6 + 2i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} &= \frac{6 + 12i + 2i + 4i^2}{1 - 4i^2} \\ &= \frac{6 + 14i - 4}{1 + 4} = \frac{2 + 14i}{5} \end{aligned}$$