NOTES: SECONDARY 2 HONORS

QUADRATIC EXPRESSIONS (5.1A, 5.1B, 5.1C)

STARTER

1. What is the standard form of a quadratic expression? $f(x)=ax^2+bx+c$	2. Factor $x^2-2x-15$ $(x+3)(x-5)$	3. Factor $6x^2-11x-2$ $(6x+1)(x-2)$
4. Factor $4x^2-9$ $(2x)^2 \quad (3)^2$ $(2x+3)(2x-3)$	5. Write the quadratic in vertex form $y=2x^2-8x+3$ $y=a(x-h)^2+k$ complete the square. $y=(2x^2-8x+\_\_)+3-\_\_$ $y=2(x^2-4x+4)+3-(2)4$ $\frac{-4}{2}=-2 \quad (-2)^2=4$ $\boxed{y=2(x-2)^2-5}$	6. Simplify $\sqrt{48x^3}$ 4 12 2 2 4 3 2 2 $2\cdot 2\lvert x\rvert\sqrt{3x} = \boxed{4\lvert x\rvert\sqrt{3x}}$

Vocabulary

- A quadratic pattern can be found in other types of expressions and equations. If this is the case, we say these expressions, equations, or functions are Quadratic in nature.

Example: Determine if the following expressions are quadratic in nature. If it is quadratic in nature, rewrite the expression in quadratic form.

a. $3x^8+4x^4-9$

yes

$3(x^4)^2+4(x^4)-9$

b. $-5x^{10}-4x^5+2$

yes

$-5(x^5)^2-4(x^5)+2$

c. $x^{\frac{1}{2}}+4x^{\frac{1}{4}}-3$

yes

$(x^{\frac{1}{4}})^2+4(x^{\frac{1}{4}})-3$

d. x^5+4x^2-2

NO

$(x^2)^2=x^4$

When factoring expressions that are quadratic in nature, it is sometimes easier to rewrite the quadratic using "u" substitution before you factor.

STEP #1: DETERMINE IF THE EXPRESSION IS QUADRATIC IN NATURE.

Example: Determine if the expression is quadratic in nature.

a. $10x^4+11x^2-6$

yes

$10(x^2)^2+11(x^2)-6$

b. $3x^{\frac{1}{3}}-8x^{\frac{1}{6}}+4$

yes

$3(x^{\frac{1}{6}})^2-8(x^{\frac{1}{6}})+4$

c. $(x+1)^4-2(x+1)^2-15$

yes

$((x+1)^2)^2-2(x+1)^2-15$

STEP #2: IDENTIFY WHAT "u =" THAT WOULD ALLOW YOU TO REWRITE THE EXPRESSION AS A QUADRATIC.

a. $10x^4 + 11x^2 - 6$

$u = x^2$

b. $3x^{\frac{1}{3}} - 8x^{\frac{1}{6}} + 4$

$u = x^{\frac{1}{6}}$

c. $(x+1)^4 - 2(x+1)^2 - 15$

$u = (x+1)^2$

STEP #3: SUBSTITUTE "u =" INTO THE EXPRESSION.

a. $10x^4 + 11x^2 - 6$

$10u^2 + 11u - 6$

b. $3x^{\frac{1}{3}} - 8x^{\frac{1}{6}} + 4$

$3u^2 - 8u + 4$

c. $(x+1)^4 - 2(x+1)^2 - 15$

$u^2 - 2u - 15$

STEP #4: FACTOR THE EXPRESSION USING THE DIFFERENT QUADRATIC FACTORING TECHNIQUES (GCF, difference of two squares, perfect square trinomial, "ac method", guess and check)

a. $10x^4 + 11x^2 - 6$

$10u^2 + 15u - 4u - 6$

$5u(2u+3) - 2(2u+3)$

$= (2u+3)(5u-2)$

b. $3x^{\frac{1}{3}} - 8x^{\frac{1}{6}} + 4$

$(3u-2)(u-2)$

c. $(x+1)^4 - 2(x+1)^2 - 15$

$(u+3)(u-5)$

STEP #5: REPLACE THE "u".

a. $10x^4 + 11x^2 - 6$

$(2x^2+3)(5x^2-2)$

b. $3x^{\frac{1}{3}} - 8x^{\frac{1}{6}} + 4$

$(3x^{\frac{1}{6}}-2)(x^{\frac{1}{6}}-2)$

c. $(x+1)^4 - 2(x+1)^2 - 15$

$\boxed{((x+1)^2+3)((x+1)^2-5)}$

Now you try....

1. $8x^6 + 2x^3 - 15$

$u = x^3$

$8u^2 + 2u - 15$

$(4u-5)(2u+3)$

$\boxed{(4x^3-5)(2x^3+3)}$

2. $100x^8 - 121y^6$

$(10x^4)^2 - (11y^3)^2$

$\boxed{(10x^4 + 11y^3)(10x^4 - 11y^3)}$

3. $4x^4 - 20x^2 + 25$

$u = x^2$

$4u^2 - 20u + 25$

$(2u-5)^2$

$\boxed{(2x^2-5)^2}$

4. $9x^{10} - 6x^5 y + y^2$

$2(3x^5)(y) = 6x^5 y$

$\boxed{(3x^5 - y)^2}$

5. $12x^{\frac{2}{5}} - 17x^{\frac{1}{5}} + 6$

$u = x^{1/5}$

$12u^2 - 17u + 6$

$(3u-2)(4u-3)$

$\boxed{(3x^{\frac{1}{5}}-2)(4x^{\frac{1}{5}}-3)}$

6. $3x^{\frac{2}{3}} + 10x^{\frac{1}{3}} + 8$

$u = x^{1/3}$

$3u^2 + 10u + 8$

$(3u+4)(u+2)$

$\boxed{(3x^{\frac{1}{3}}+4)(x^{\frac{1}{3}}+2)}$

7. $81x^6 - 4$

$(9x^3)^2 - (2)^2$

$\boxed{(9x^3-2)(9x^3+2)}$

8. $2x - \sqrt{x} - 1$

$(\sqrt{x})^2 = x$

$(x^{\frac{1}{2}})^2 = x$

$2x - x^{\frac{1}{2}} - 1$

$u = x^{\frac{1}{2}}$

$2u^2 - u - 1$

$(2u+1)(u-1)$

$\boxed{(2\sqrt{x}+1)(\sqrt{x}-1)}$