NOTES: SECONDARY 2 HONORS
QUADRATIC FORMULA (5.3D/5.3E)

STARTER

1. What is the standard form of a quadratic expression?

ax^2+bx+c

2. Simplify $x=\dfrac{-4\pm\sqrt{36}}{2}$

$x=\dfrac{-4\pm 6}{2}=-2\pm 3$

$x=-2+3$ $\quad \boxed{x=1}$

$x=-2-3$ $\quad \boxed{x=-5}$

3. Simplify $x=\dfrac{-4\pm\sqrt{-36}}{2}$ $\quad i\sqrt{36}$

$x=\dfrac{-4\pm 6i}{2}$ $\quad \boxed{x=-2\pm 3i}$

$x=-2+3i \quad x=-2-3i$

4. Simplify $x=\dfrac{12\pm\sqrt{-18}}{6}$ $:\ i\sqrt{18}$

$x=\dfrac{12\pm 3i\sqrt{2}}{6}$

$\boxed{x=\dfrac{4\pm i\sqrt{2}}{2}}$

5. Solve the quadratic by completing the square.

$0=-3x^2-12x+2$

$0=(-3x^2-12x+\_\_\_)+2-\_\_\_$

$0=-3(x^2+4x+4)+2-(-3)4$

$0=-3(x+2)^2+14$

$-14 \qquad -14$

$\dfrac{-14}{-3}=\dfrac{-3(x+2)^2}{-3}$

$\sqrt{\dfrac{14}{3}}=\sqrt{(x+2)^2}$

$\pm\sqrt{\dfrac{14}{3}}=x+2$

$\pm\dfrac{\sqrt{14}}{\sqrt{3}}=x+2$

$\boxed{x=-2\pm\dfrac{\sqrt{14}}{\sqrt{3}}}$

So far we have discussed how to solve a quadratic formula by factoring or complete the square. There is one more method we can use. The quadratic formula allows us to solve any quadratic formula.

DERIVE THE QUADRATIC FORMULA:

$ax^2+bx+c=0$

$(ax^2+bx+\_\_\_)+c-\_\_\_=0$ $\quad \dfrac{ab^2}{4a^2}=\dfrac{b^2}{4a}$

$a(x^2+\frac{b}{a}x+\frac{b^2}{4a^2})+c-(a)\frac{b^2}{4a^2}=0$

$\dfrac{\frac{b}{a}}{2}=\dfrac{b}{a}\cdot\dfrac{1}{2}=\dfrac{b}{2a}$ $\quad \left(\dfrac{b}{2a}\right)^2=\dfrac{b^2}{4a^2}$

$a\left(x+\frac{b}{2a}\right)^2+\frac{c}{1}-\frac{b^2}{4a}=0$

$-c+\frac{b^2}{4a} \qquad -c+\frac{b^2}{4a}$

$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-\frac{c}{1}\cdot\frac{4a}{4a}=\frac{b^2-4ac}{4a}$

$\left(\frac{1}{a}\right)a\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}\left(\frac{1}{a}\right)$

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}=\frac{\pm\sqrt{b^2-4ac}}{2a}$

$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$

$-\frac{b}{2a} \qquad -\frac{b}{2a}$

$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve the quadratic equation and identify the solutions as real or imaginary.

1. $3x^2 + 5x - 4 = 0$

2. $2x^2 - 3 = x$ $\quad a=2 \quad b=-1 \quad c=-3$

$2x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1-4(2)(-3)}}{2(2)} = \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm \sqrt{25}}{4}$

$x = \frac{1 \pm 5}{4}$

$x = \frac{1+5}{4}$ $\quad x = \frac{1-5}{4}$

$x = 6/4$ $\quad x = \frac{-4}{4}$

$\boxed{x = \frac{3}{2}}$ $\quad \boxed{x = -1}$

3. $1 + 10x = -25x^2$

4. $2x^2 + 12x + 20 = 0$

$a=2 \quad b=12 \quad c=20$

$x = \frac{-12 \pm \sqrt{144 - 4(2)(20)}}{2(2)} = \frac{-12 \pm \sqrt{144-160}}{4} = \frac{-12 \pm \sqrt{-16}}{4}$

$x = \frac{-12 \pm 4i}{4}$ $\quad \boxed{x = -3 \pm i}$

What do you know about the solution of a quadratic equation……

- if the value under the square root sign *(discriminant)* is negative? 2 imaginary solutions
- if the value under the square root sign *(discriminant)* is positive? 2 real solutions
- if the value under the square root sign *(discriminant)* is zero? 1 real solution

Vocabulary:

- The discriminant tells us about the nature of the solutions for a quadratic equation. The discriminant is the value under the square root sign of the formula.

If $b^2 - 4ac > 0$,	then there are two Real solutions to a quadratic equation, which means the parabola crosses the *x*-axis twice.
If $b^2 - 4ac = 0$,	then there is one Real solution to a quadratic equation, which means the vertex is on the *x*-axis.
If $b^2 - 4ac < 0$,	then there are no Real, but two Imaginary solutions to a quadratic equation, which means the parabola does NOT cross the *x*-axis.

Example: Calculate the discriminant for each quadratic and describe the nature of the roots. Find the zeros.

1. $5x + 5 = -x^2$

$x^2 + 5x + 5 = 0$

Discriminant: $b^2 - 4ac$

$= 5^2 - 4(1)(5)$

$= 25 - 20 = 5$

Discriminant: 5
2 real solution

$\boxed{x = \frac{-5 \pm \sqrt{5}}{2}}$

2. $9x^2 + 12x + 13 = 0$

Discrimant: $b^2 - 4ac$

$144 - 4(9)(13)$

$= 144 - 468$

$= -324$

Discriminant: -324
2 imaginary solutions

$x = \frac{-12 \pm \sqrt{-324}}{2(9)} = \frac{-12 \pm 18i}{18}$

$\boxed{x = \frac{-2 \pm 3i}{3}}$