Secondary 2H
Unit 5

Name______________ **Desk** ________

Unit 5 STUDY GUIDE

Score ________

The TEST will be on: MONDAY NOV. 25

Skills you should have mastered:

- Identify if an expression is quadratic in nature. Explain your reasoning in a complete sentence.
- Factor an expression that is quadratic in nature using "u" substitution.
- Simplify complex expressions. (Addition, subtraction, multiplying, and dividing)
- Find the conjugate of a complex number
- Graph a complex number on the complex plane and find the modulus.
- Find the discriminant and use the discriminant to determine the nature of the zeros.
- Find the zeros of a quadratic by the quadratic formula.
- Solve a quadratic equation using one of the five methods. (Choose the method that best fits the problem.)
- Concepts from Units 1-4. (These are fair game!!)

Determine if the quadratic expression is "quadratic in nature". Explain your reasoning in a complete sentence.

1. $-5x^{10} - 4x^5 + 2$

Yes, because...

2. $x^5 + 4x^2 - 2$

No, because...

3. $x^3 - 4x + 4$

No, because...

4. $(x+1)^4 - 2(x+1) - 15$

No, because...

5. $x^{\frac{1}{2}} + 4x^{\frac{1}{4}} - 3$

yes, because

6. $9x^{\frac{2}{3}} - 12x^{\frac{1}{3}} + 4$

Yes, because...

Factor the following polynomials using "u" substitution.

1. $8x^6 + 2x^3 - 15$ $u = x^3$

$8u^2 + 2u - 15$

$(2u+3)(4u-5)$

$\boxed{(2x^3+3)(4x^3-5)}$

2. $100x^8 - 121y^6$ $u = x^4$, $w = y^3$

$100u^2 - 121w^2$

$= (10u - 11w)(10u + 11w)$

$= \boxed{(10x^4 - 11y^3)(10x^4 + 11y^3)}$

3. $4x^4 - 20x^2 + 25$ $u = x^2$

$4u^2 - 20u + 25$

$(2u-5)^2$

$= \boxed{(2x^2-5)^2}$

4. $9x^{10} - 6x^5y + y^2$ $u = x^5$

$9u^2 - 6uy + y^2$

$(3u - y)^2$

$= \boxed{(3x^5 - y)^2}$

5. $2x - \sqrt{x} - 1$ $u = x^{\frac{1}{2}}$

$2x - x^{\frac{1}{2}} - 1$

$2u^2 - u - 1$

$(2u+1)(u-1)$

$= \boxed{(2x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)}$

or

$(2\sqrt{x}+1)(\sqrt{x}-1)$

6. $2(2x-3)^2 + (2x-3) - 1$

$u = (2x-3)$

$2u^2 + u - 1$

$(2u-1)(u+1)$

$= \boxed{(2(2x-3)-1)(2(2x-3)+1)}$

Simplify the following expressions. Remember that *i* should never be raised to a power or appear in the denominator of a fraction.

1. $-\sqrt{-50}$

$25 \quad 2$

$5 \quad 5$

$\boxed{-5i\sqrt{2}}$

2. $\sqrt{-28}$

$4 \quad 7$

$2 \quad 2$

$\boxed{2i\sqrt{7}}$

3. $(4-5i)-(8-3i)$

$\boxed{-4-2i}$

4. $(4-5i)+(3-6i)$

$= \boxed{7-11i}$

5. $(3-5i)(2+3i)$

$= 6+9i-10i-15i^2$

$= 6-i+15$

$= \boxed{21-i}$

6. $(11+2i)(3-7i)$

$= 33-77i+6i-14i^2$

$= \boxed{47-71i}$

7. i^{19}

$\frac{19}{4} = 4 \text{ R } 3$

$i^{19} = i^3 = \boxed{-i}$

8. i^{297}

$\frac{297}{4} = 74 \text{ R } 1$

$i^{297} = i^1 = \boxed{i}$

9. i^{32767}

$\frac{32767}{4} = 8191 \text{ R } 3$

$i^{32767} = i^3 = \boxed{-i}$

10. What is the conjugate of $6+6i$?

$\boxed{6-6i}$

11. What is the conjugate of $-2+3i$?

$\boxed{-2-3i}$

12. $\frac{-8-3i}{-2i} \cdot \frac{2i}{2i}$

$= \frac{-16i-6i^2}{-4i^2} = \frac{6-16i}{4}$

$= \boxed{\frac{3-8i}{2}}$

13. $\frac{8-3i}{4+2i} \cdot \frac{4-2i}{4-2i}$

$= \frac{32-16i-12i+6i^2}{16-8i+8i-4i^2}$

$= \frac{32-28i-6}{16+4}$

$= \frac{26-28i}{20} = \boxed{\frac{13-14i}{10}}$

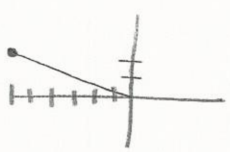
14. $\frac{6i}{-3+i} \cdot \frac{-3-i}{-3-i}$

$= \frac{-18i-6i^2}{9+3i-3i-i^2} = \frac{6-18i}{9+1}$

$= \frac{6-18i}{10} = \boxed{\frac{3-9i}{5}}$

Draw the complex number on the complex plane and then find the modulus.

1. $-6+2i$



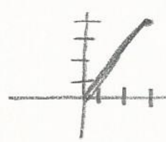
$|-6+2i|$

$= \sqrt{(-6)^2+(2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40} = \boxed{2\sqrt{10}}$

2. $3+4i$



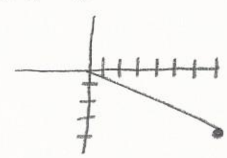
$|3+4i|$

$= \sqrt{(3)^2+(4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = \boxed{5}$

3. $8-4i$



$|8-4i|$

$= \sqrt{(8)^2+(4)^2}$

$= \sqrt{64+16}$

$= \sqrt{80} = \boxed{4\sqrt{5}}$

Derive the quadratic formula from the standard form of a quadratic equation.

$ax^2+bx+c=0$

$(ax^2+bx+\_\_\_)+c-\_\_\_=0$

$a(x^2+(\frac{b}{a})x+\_\_\_)+c-(a)\_\_\_=0$

$\frac{\frac{b}{a}}{2} = \frac{b}{2a} \qquad (\frac{b}{2a})^2 = \frac{b^2}{4a^2}$

$a(x^2+\frac{b}{a}x+\frac{b^2}{4a^2})+c-(a)(\frac{b^2}{4a^2})=0$

$a(x+\frac{b}{2a})^2+c-\frac{b^2}{4a}=0$

$a(x+\frac{b}{2a})^2+c-\frac{b^2}{4a}=0$

$-c+\frac{b^2}{4a} \qquad -c+\frac{b^2}{4a}$

$\frac{a(x+\frac{b}{2a})^2}{a} = \frac{\frac{b^2}{4a}-c}{a} = \frac{\frac{b^2-4ac}{4a}}{a}$

$\sqrt{(x+\frac{b}{2a})^2} = \sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{\sqrt{4a^2}} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$-b/2a \qquad -b/2a$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac Quadratic Formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Calculate the discriminant for each quadratic and describe the nature of the roots. Then, find the zeros.

1. $2x^2+4x=2$

$2x^2+4x-2=0$

$a=2$ $b=4$ $c=-2$

Discriminant: b^2-4ac

$=(4)^2-4(2)(-2)$

$=16+16=32$

$\boxed{\text{Discriminant}=32 \text{ 2 real zeros}}$

$x=\frac{-4\pm\sqrt{32}}{2(2)}=\frac{-4\pm4\sqrt{2}}{4}$

$\boxed{x=-1\pm\sqrt{2}}$

2. $-x^2+2x-5=0$

$a=-1$ $b=2$ $c=-5$

$b^2-4ac=(2)^2-4(-1)(-5)$

$=4-20=-16$

$\boxed{\text{Discriminant: }-16 \text{ 2 imaginary zeros}}$

$x=\frac{-2\pm\sqrt{-16}}{2(-1)}=\frac{-2\pm4i}{-2}$

$\boxed{x=1\pm2i}$

3. $4x^2+12x+9=0$

$a=4$ $b=12$ $c=9$

$b^2-4ac=(12)^2-4(4)(9)$

$=144-144$

$=0$

$\boxed{\text{Discriminant: }0 \text{ 1 real zero}}$

$x=\frac{-12\pm\sqrt{0}}{2(4)}=\frac{-12}{8}$

$\boxed{x=\frac{-3}{2}}$

4. $x^2+8x+12=0$

$a=1$ $b=8$ $c=12$

$b^2-4ac=(8)^2-4(1)(12)$

$=64-48$

$=16$

$\boxed{\text{Discriminant: }16 \text{ 2 real zeros}}$

$x=\frac{-8\pm\sqrt{16}}{2(1)}=\frac{-8\pm4}{2}$

$x=\frac{-8+4}{2}=\frac{-4}{2}$ $x=\frac{-8-4}{2}=\frac{-12}{2}$

$\boxed{x=-2}$ $\boxed{x=-6}$

There are five different methods used to solve quadratic equations:

1. Square Root Method
2. Factoring Method
3. Completing the Square Method
4. Quadratic Formula Method
5. Graphing Method (from Unit 2)

1. $4x^2+81=0$

$-81\ -81$

$\frac{4x^2}{4}=\frac{-81}{4}$

$\sqrt{x^2}=\sqrt{\frac{-81}{4}}=\frac{\sqrt{-81}}{\sqrt{4}}$

$\boxed{x=\pm\frac{9}{4}i}$

2. $(x-1)^2-5=44$

$+5\ +5$

$\sqrt{(x-1)^2}=\sqrt{49}$

$x-1=\pm\sqrt{49}$

$x-1=\pm7$

$x=-1\pm7$

$x=-1+7$ $x=-1-7$

$\boxed{x=6}$ $\boxed{x=-8}$

3. $x^2+4x+6=0$

$(x^2+4x+\_\_)+6-\_\_=0$

$(x^2+4x+4)+6-4=0$

$(x+2)^2+2=0$

$(x+2)^2=-2$

$x+2=\pm i\sqrt{2}$

$\boxed{x=-2\pm i\sqrt{2}}$

4. $4x^2+4x=3$

$4x^2+4x-3=0$

$(2x+3)(2x-1)=0$

$2x+3=0$ $2x-1=0$

$\boxed{x=\frac{-3}{2}}$ $\boxed{x=\frac{1}{2}}$

5. $x^2+4=0$

$-4\ -4$

$x^2=-4$

$\boxed{x=\pm2i}$

6. $2x^2-5x-4=0$

$a=2$ $b=-5$ $c=-4$

$x=\frac{5\pm\sqrt{25-4(2)(-4)}}{2(2)}$

$x=\frac{5\pm\sqrt{25+32}}{4}$

$=\boxed{\frac{5\pm\sqrt{57}}{4}}$

7. $x^2+x-30=0$

$(x+6)(x-5)=0$

$x+6=0$ $x-5=0$

$\boxed{x=-6}$ $\boxed{x=5}$

8. $2(x+3)^2+5=23$

$-5\ -5$

$\frac{2(x+3)^2}{2}=\frac{18}{2}$

$\sqrt{(x+3)^2}=\sqrt{9}$

$x+3=\pm9$

$x=-3+9$ $x=-3-9$

$\boxed{x=6}$ $\boxed{x=-12}$