

HW: Independent Events

SHOW ALL WORK. If you need to show your work on a separate piece of paper, please do!

1. The table below shows four sets of values for $P(A)$, $P(B)$, $P(A \cap B)$. Based on the definition of independence, determine if events A and B are independent in each case.

| | $P(A)$ | $P(B)$ | $P(A \cap B)$ | Are A and B independent? (yes/no) |
|----|---------------|---------------|----------------|-----------------------------------|
| a. | 0.2 | 0.14 | 0.028 | |
| b. | 0.32 | 0.16 | 0.48 | |
| c. | $\frac{1}{3}$ | $\frac{3}{5}$ | $\frac{4}{15}$ | |
| d. | $\frac{7}{8}$ | $\frac{2}{5}$ | $\frac{7}{20}$ | |

2. Paola is playing a word game in which she draws letter tiles from a bag without looking. The bag contain 7 tiles: 2 As, 3 Es, and 2 Rs. For each of problems a and b, find the probability of getting an E first and getting an E second. In each problem, state whether the events are independent.

a. Paola takes a tile, then replaces it, and then takes a second tile.

b. Paola takes a tile, does not replace it, and then takes a second tile.

3. Suppose that a married couple will have 3 children and suppose that having a girl or boy is equally likely each time.

Consider the following events.

$$S = \{GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB\}$$

A: At least 2 consecutive children are of the same gender.

B: Exactly 2 consecutive children are of the same gender.

C: No 2 consecutive children are of the same gender.

For each pair of events, determine if the events are independent.

$A \cap B$ $P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$
 $\frac{4}{8} \neq \frac{6}{8} \cdot \frac{4}{8}$ NOT INDEPENDENT

$A \cap C$ $P(A \cap C) \stackrel{?}{=} P(A) \cdot P(C)$
 $0 \neq \frac{6}{8} \cdot \frac{2}{8}$ NOT INDEPENDENT

$B \cap C$ $P(B \cap C) \stackrel{?}{=} P(B) \cdot P(C)$
 $0 \neq \frac{4}{8} \cdot \frac{2}{8}$ NOT INDEPENDENT

4. The Coolest Deal is a daily special sold at Ike's Ice Cream Parlor. One day, the Coolest Deal is a large cone with one topping. The following table shows the sales data for the Coolest Deal that day.

Coolest Deals Sold at Ike's

| Topping choice | Ice cream choice | | | |
|----------------|------------------|-----------|--------------|-----------|
| | Vanilla | Chocolate | Cookie dough | Mint chip |
| Sprinkles | 9 | 12 | 16 | 14 |
| Hot fudge | 11 | 4 | 16 | 15 |
| Caramel | 10 | 12 | 18 | 15 |

Using the data in the table, determine if the events stated in problems a and b seem to be independent. Show the work that supports your answer.

a. A random customer at Ike's orders caramel and cookie dough for the Coolest Deal.

C: caramel

D: cookie dough

$$P(C \cap D) \stackrel{?}{=} P(C) \cdot P(D)$$

$$\frac{18}{152} = \frac{55}{152} \cdot \frac{50}{152}$$

independent

$$0.118 \approx 0.119$$

b. A random customer at Ike's orders hot fudge and chocolate for the Coolest Deal.

H: hot fudge

C: chocolate

$$P(H \cap C) \stackrel{?}{=} P(H) \cdot P(C)$$

$$\frac{4}{152} \neq \frac{46}{152} \cdot \frac{28}{152}$$

NOT INDEPENDENT

$$0.026 \neq 0.056$$

5. For a statistics project, Tamara surveys a well-chosen sample that represents all the students at her school. She finds that 72% have at least one sibling (brother or sister) and 27% have at least one sibling and at least one pet in their home. Assume that having a sibling and having a pet are independent events. Based on the survey, what is the probability that a randomly chosen student at Tamara's school has at least one pet at home?

S: sibling

P: pets

$$P(S) = 72\%$$

$$P(P) = ?$$

$$P(P \cap S) = 27\%$$

6. Emily and Nino are participating in an archery unit in their physical education class. Emily has hit inside the yellow region 16 times out of 40 shots. Nino has hit inside the yellow region 15 times out of 30 shots. They are now partners in a "take your best shot" tournament. In the tournament, each partner shoots once and the best shot counts. Assuming that Emily's result and Nino's result are independent of each other, what is the probability that Emily or Nino will hit inside the yellow region, based on past performance

↑
 To find "or" we need to use the Addition Rule.

E: Emily
 N: Nino

$$P(E \cup N) = P(E) + P(N) - P(E \cap N)$$

↑
 Assuming E and N are independent events, can I find $P(E \cap N)$?

7. Mario's job evaluation has two components: punctuality (arriving on time) and task performance. To be rated satisfactory overall, he needs to be rated satisfactory for both components. His record shows the following: rated satisfactory overall 76% of work days and rated satisfactory in task performance 80% of work days. Assume that arriving on time and task performance are independent in Mario's case. What is the probability that he will arrive on time on his next work day, based on the data.

8. At Louie's Book Café, every sale is recorded as either "reading" or "café". Last month, 67.5% of people who visited Louie's spent money there, and 50% of the visitors made at least one café purchase. Assume that buying a reading item and buying a café item are independent events. What is the probability that the next visitor at Louie's will buy a reading item, based on last month's data? (Hint: Use the Addition Rule.)

R: reading
 C: café

$$P(R \cup C) = P(R) + P(C) - P(R \cap C)$$

*When using the Addition Rule, write each probability as a decimal and not a percent.

9. Three events A, B, and C, are independent if the following conditions are all satisfied:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap C) = P(A) \cdot P(C)$$

$$P(B \cap C) = P(B) \cdot P(C)$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

Consider these coin-tossing experiments and associated events:

Experiment 1: Toss a coin 2 times.

- Event A: Get a heads on the first toss.
- Event B: Get heads on the second toss.
- Event C: Get exactly one head.

$$S = \{HH, HT, TH, TT\}$$

Experiment 2: Toss a coin 3 times.

- Event A: Get heads on the first toss.
- Event B: Get heads on the second toss.
- Event C: Get heads on the third toss.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

For each experiment, determine whether events A, B, and C are independent.

EXPERIMENT 1:

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{2} \quad P(C) = \frac{1}{2} \quad P(A \cap B) = \frac{1}{4} \quad P(A \cap C) = \frac{1}{4} \quad P(B \cap C) = \frac{1}{4} \\ P(A \cap B \cap C) = 0$$

- Does $P(A \cap B) = P(A) \cdot P(B)$?
- Does $P(A \cap C) = P(A) \cdot P(C)$?
- Does $P(B \cap C) = P(B) \cdot P(C)$?
- Does $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$?

All four of these statements must be true for A, B and C to be independent.

EXPERIMENT 2:

Same idea as experiment 1.