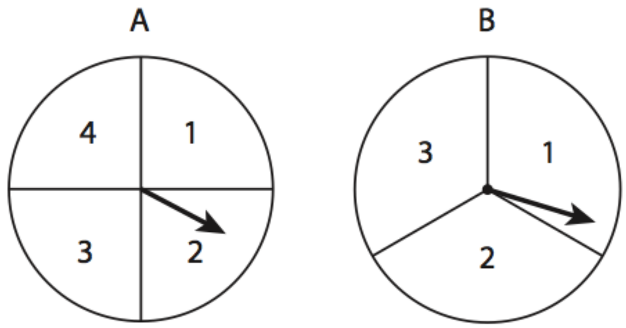
NOTES: MATH 2 HONORS
Unit 7: Conditional Probability

Example 1: Howard is playing a carnival game. The object of the game is to predict the sum you will get by spinning spinner A and then spinner B. Howard predicts he will get a sum of 5.



1. List the sample space.

(1,1) (2,1) (3,1) (4,1)
(1,2) (2,2) (3,2) (4,2)
(1,3) (2,3) (3,3) (4,3)

→ 12 outcomes

2. What is the probability that Howard gets a sum of 5? Explain.

$\frac{3}{12} = \boxed{\frac{1}{4}}$

3. Suppose Howard gets a 3 spinner A. Now what is the probability that Howard gets a sum of 5? Explain.

(3,1) (3,2) (3,3)

$\boxed{\frac{1}{3}}$

4. How does getting a 3 on spinner A affect the probability that Howard gets a sum of 5? Explain.

Getting a 3 on spinner A increases the probability that Howard will get a sum of 5 because $\frac{1}{3} > \frac{1}{4}$

Recall: Events are **dependent** if the occurrence of one changes the probability of another event occurring. For example, drawing marbles from a bag with replacement is independent, while drawing marbles from a bag without replacement is dependent.

The **conditional probability of *B* given *A*** is the probability that event *B* occurs, given that event *A* has already occurred and is denoted as $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

If *A* and *B* are independent, then the formula for $P(A \text{ and } B)$ is the equation used in the definition of independent events:

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ formula for $P(A \text{ and } B)$
$P(A \text{ and } B) = P(A) \cdot P(B)$ formula for $P(A \text{ and } B)$ if A and B are independent

$P(B) = P(B|A)$

Example 2: Alexis rolls a pair of number cubes. What is the probability that both numbers are odd if their sum is 6?

A: both numbers are odd

B: The sum of the numbers is 6.

What is the probability that both numbers are odd given that their sum is 6? $P(A|B)$

$S=$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$6 \cdot 6 = 36$

A: $\frac{9}{36}$

B: $\frac{5}{36}$

$A \cap B$: $\frac{3}{36}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{3}{36}}{\frac{5}{36}} = \frac{3}{36} \cdot \frac{36}{5} = \frac{3}{5}$$

Example 3: Hamid rolls a number cube 3 times. Consider the following events.

A: The first roll is an odd number.

B: There are exactly 2 consecutive odd numbers.

Determine if *A* and *B* are independent events.

Sample Space: 216 outcomes $6 \cdot 6 \cdot 6$

EVEN/ODD: {OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$P(A): \frac{4}{8} = \frac{1}{2}$ $P(B) = \frac{2}{8} = \frac{1}{4}$

A|B: The first roll is odd given there are exactly 2 consecutive odd numbers.

{OOE, EOO} $P(A|B) = \frac{1}{2}$

B|A: There are exactly 2 consecutive odd numbers given the first roll is odd.

{OOO, OOE, OEO, OEE} $P(B|A) = \frac{1}{4}$

$P(A|B) \stackrel{?}{=} P(A)$ $P(B|A) \stackrel{?}{=} P(B)$

$\frac{1}{2} = \frac{1}{2}$ ✓ $\frac{1}{4} = \frac{1}{4}$ ✓

yes, they are independent.

CHECK: $P(A\cap B) \stackrel{?}{=} P(A) \cdot P(B)$

Example 4: A vacation resort offers bicycles and personal watercrafts for rent. The resort's manager made the following notes about rentals:

- 200 customers rented items in all – 100 rented bicycles and 100 rented personal watercrafts.
- Of the personal watercraft customers, 75 customers were young (30 years old or younger) and 25 customers were older (31 years old or older).
- 125 of the 200 customers were age 30 or younger. 50 of these customers rented bicycles, and 75 of them rented personal watercrafts.

Consider the following events that apply to a random customer.

Y: The customer is young (30 years old or younger)

W: The customer rents a personal watercraft.

Are *Y* and *W* independent? Compare $P(Y|W)$ and $P(W|Y)$ and interpret the results.

$P(Y) = \frac{125}{200} = 0.625$

$P(W) = \frac{100}{200} = 0.5$

$P(Y|W) = \frac{75}{100} = 0.75$

$P(W|Y) = \frac{75}{125} = 0.6$

$P(Y|W) \neq P(Y)$

$P(W|Y) \neq P(W)$

Y and W are not independent.

$P(Y|W) = 0.75 > P(W|Y) = 0.6$

This means that it is more likely that a customer is young given they rented a personal watercraft.

Example 5: Last year at All Technical High School, 20% of the students received an academic award. That same year, 16% of the students at the school received an academic award and a service award. What is the probability that a student who receives an academic award also receives a service award?

What is the probability that a student receives a service award given that they earned an academic award?

A: academic award B: service award

Find $P(B|A)$.

$P(A) = 0.2$ $P(B \cap A) = 0.16$

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.16}{.2}$ $P(B|A) = .8$

There is an 80% probability that a student receives an academic award and also a service award.