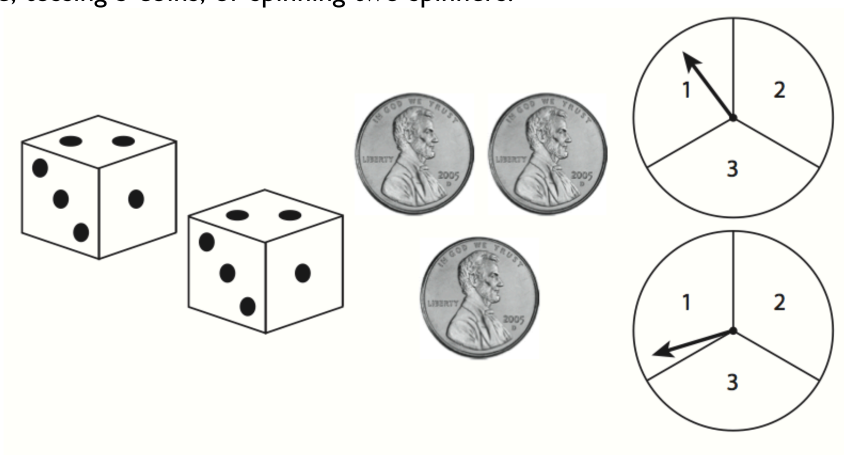
NOTES: MATH 2 HONORS
Unit 7: Independent Events

Example 1: Tonya's class is studying probability by playing games, or conducting probability experiments. Tonya can choose one of three experiments. If she gets an outcome with all matching results, she wins a prize. The experiments are rolling a pair of dice, tossing 3 coins, or spinning two spinners.



Which experiment should Tonya choose to have the best chance at winning the prize? Explain your reasoning.

Dice: $\frac{6}{36} = \frac{1}{6}$

coins: $\frac{2}{8} = \frac{1}{4}$

spinners: $\frac{3}{9} = \frac{1}{3}$

Tonya should choose the spinners because $\frac{1}{3}$ is greater than $\frac{1}{6}$ and $\frac{1}{4}$.

Two events are **independent** if the occurrence or non-occurrence of one event has no effect on the probability of the other even. If two events are independent, then you can simply multiply their individual probabilities to find the probability that both events will occur. If events are **dependent**, then the outcome of one event affects the outcome of another event. So it is important to know whether or not two events are independent.

Two events *A* and *B* are independentdent if and only if $P(A\ and\ B) = P(A) \cdot P(B)$ or $P(A \cap B) = P(A) \cdot P(B)$.

Example 2: Ms. Martinez is teaching her students about probability. She has a bag of 6 marbles; 2 are red and 4 are black. She asked her students to conduct the following experiments:

Experiment A: Pick a random marble from the bag, then *replace it*, and then pick a random marble from the bag again.

Experiment B: Pick a random marble from the bag, *do not replace it*, and then pick a random marble from the bag again.

For each experiment, determine whether getting red on the first pick and getting black on the second pick are independent events.

EXPERIMENT A: INDEPENDENT

EXPERIMENT B: DEPENDENT

Example 3: Trevor tosses a coin 3 times. Consider the following events.

A: The first toss is heads.

B: The second toss is heads.

C: There are exactly 2 consecutive heads.

For each of the following pairs of events, determine if the events are independent.

$A \cap B$

$A \cap C$

$B \cap C$

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$P(A) = \frac{4}{8}$ $P(B) = \frac{4}{8}$ $P(C) = \frac{2}{8}$

$P(A \cap B) = \frac{2}{8}$ $P(A \cap C) = \frac{1}{8}$ $P(B \cap C) = \frac{2}{8}$

$P(A \cap B) = P(A) \cdot P(B)$ $P(A \cap C) = P(A) \cdot P(C)$ $P(B \cap C) = P(B) \cdot P(C)$

$\frac{2}{8} = \frac{4}{8} \cdot \frac{4}{8}$ $\frac{1}{8} = \frac{4}{8} \cdot \frac{2}{8}$ $\frac{2}{8} = \frac{4}{8} \cdot \frac{2}{8}$

Yes, independent — Yes, independent — No, not independent → Dependent

Example 4: Landen owns a delicatessen. He collected data on sales of his most popular sandwiches for one week and recorded it in the table below.

Sandwiches Sold in One Week

Bread choice	Sandwich choice				
	Landen's club	Turkey melt	Roasted chicken	Veggie delight	
Country white	44	25	25	8	102
Whole wheat	24	28	26	34	112
Sourdough	24	27	24	31	106
	92	80	75	73	320

Each of the following statements describes a pair of events. For each statement, determine if the events seem to be independent based on the data in the table.

- A random customer orders Landen's club sandwich on country white bread.

Landen's club: L country white bread: C

$P(L) = \frac{92}{320}$ $P(C) = \frac{102}{320}$ $P(L \cap C) = \frac{44}{320}$

$P(L \cap C) = P(L) \cdot P(C)$

$\frac{44}{320} = \frac{92}{320} \cdot \frac{102}{320}$

$.14 = .09$

$\boxed{\text{NOT INDEPENDENT}}$

- A random customer orders the roasted chicken sandwich on whole wheat bread.

Roasted chicken: R whole wheat: W

$P(R) = \frac{75}{320}$ $P(W) = \frac{112}{320}$ $P(R \cap W) = \frac{26}{320}$

$P(R \cap W) = P(R) \cdot P(W)$

$\frac{26}{320} = \frac{75}{320} \cdot \frac{112}{320}$

$.08 = .08$

$\boxed{\text{INDEPENDENT}}$

Example 5: The Town Cinema is next to Rhiannon's Bistro. A consultant surveyed 200 people who saw a particular movie at the Town Cinema and also ate at Rhiannon's Bistro. The survey indicated that 80% liked the movie and 60% liked both the movie and their meal at Rhiannon's Bistro Assume that liking the movie and liking a meal at Rhiannon's Bistro are independent events. Based on the survey, what is the probability that a randomly chosen moviegoer will like a meal at Rhiannon's Bistro?

Movie : M   Rhiannon's Bistro : R

$P(M) = 80\%$ $P(R) = ?$ $P(M \cap R) = 60\%$

$P(M \cap R) = P(M) \cdot P(R)$

$\frac{.6}{.8} = \frac{.8\,P(R)}{.8}$

$P(R) = \frac{.6}{.8}$

$\boxed{P(R) = .75 = 75\%}$

Example 6: Joel is in a basketball game. He has just tied the game. He was fouled while shooting, so he is awarded a free throw. Moreover, the referee calls a technical foul on the other team because of bad behavior, so Joel's team is awarded a second free throw. Joel has made 32 of 50 free throws so far this season, and Rico has made 42 of 48. What is the probability that Joel or Rico will made a free throw to win the game?

Joel: J   Rico: R

Can we assume the events are independent? YES

$P(J) = \frac{32}{50}$ $P(R) = \frac{42}{48}$ $P(J \cap R) = \frac{32}{50} \cdot \frac{42}{48} = 0.56$

$P(J \cup R) = P(J) + P(R) - P(J \cap R)$

$P(J \cup R) = \frac{32}{50} + \frac{42}{48} - 0.56$

$.64 + .875 - 0.56$

$\boxed{P(J \cup R) = 0.955 = 95.5\%}$