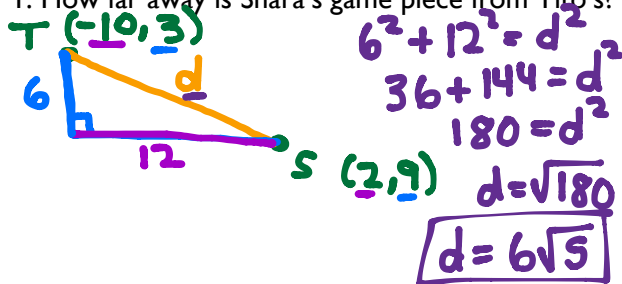


NOTES: SECONDARY 2 HONORS
UNIT 7 – DILATION AND SIMILARITY

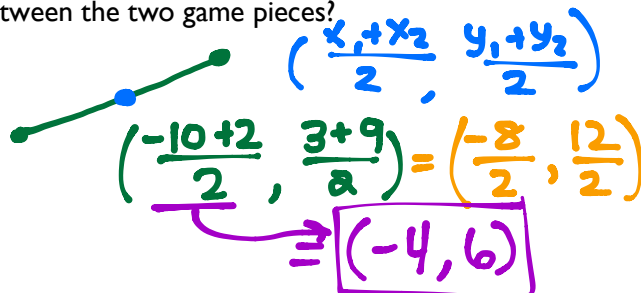
STARTER

Tito and Shara are playing a game. Tito's game piece is at the point $(-10, 3)$. Shara's game piece is at the point $(2, 9)$.

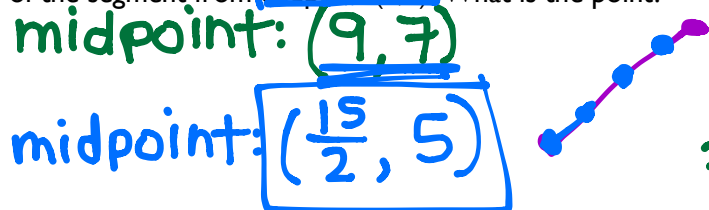
1. How far away is Shara's game piece from Tito's?



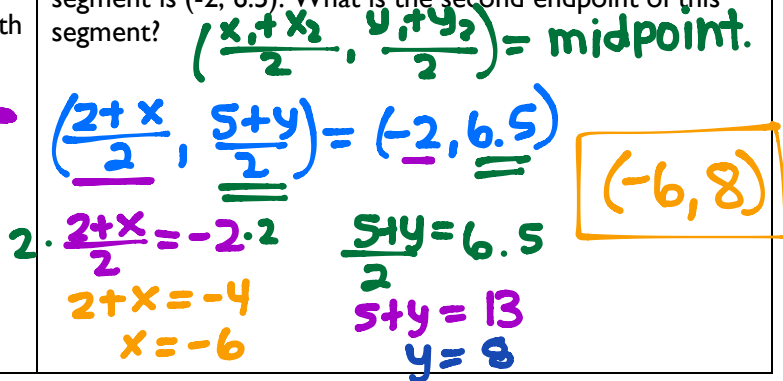
2. What are the coordinates of the point midway between the two game pieces?



3. A line segment has endpoints at $(6, 3)$ and $(12, 11)$. A point on the segment has a distance that is $\frac{1}{4}$ of the length of the segment from endpoint $(6, 3)$. What is the point?



4. One endpoint of a segment is $(2, 5)$. The midpoint of the segment is $(-2, 6.5)$. What is the second endpoint of this segment?



Essential Questions (Dilations):

1. How are the pre-image and image similar in dilations?
2. How are the pre-image and image different in dilations?
3. When are dilations used in the real world?

Essential Questions (Similarity):

1. What does it mean for two triangles to be similar?
2. How can you prove that two triangles are similar?
3. How can you use similar triangles to solve problems?

VOCAB:

- Dilation: a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point.
- Center of dilation: a point through which a dilation takes place
- Scale factor: a multiple of the lengths of the sides from one figure to the transformed figure.
 - If the scale factor is greater than 1: enlarged
 - If the scale factor is between 0 and 1: reduced
- Pre-image: the original figure prior to the transformation.
- Image: the result of a transformation.
 - The point A' is read "A prime" and is the transformed point from A point.
- Similar: two figures that are the same shape but not necessarily the same size.
 - The symbol for representing similarity between figures is ~.

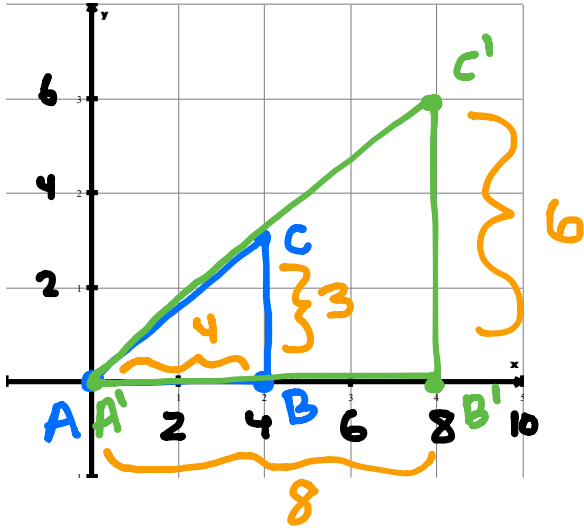
Example 1:

Given the triangle with vertices $A(0,0), B(4,0), C(4,3)$.

a. Determine the new vertices if the triangle is dilated by a scale factor of 2 and centered at $(0,0)$.

$$\begin{aligned} A(0,0) &\rightarrow A'(0,0) \\ B(4,0) &\rightarrow B'(8,0) \\ C(4,3) &\rightarrow C'(8,6) \end{aligned}$$

b. Draw $\triangle ABC$ and $\triangle A'B'C'$ on the same plane and label the vertices.



c. Verify that the ratios of corresponding parts are equal to the scale factor.

$$\text{Scale factor} = \frac{\text{length-image}}{\text{length-preimage}}$$

$$\frac{|A'B'|}{|AB|} = \frac{8}{4} = 2$$

$$\frac{|B'C'|}{|BC|} = \frac{6}{3} = 2$$

$$\frac{|A'C'|}{|AC|} = \frac{10}{5} = 2$$

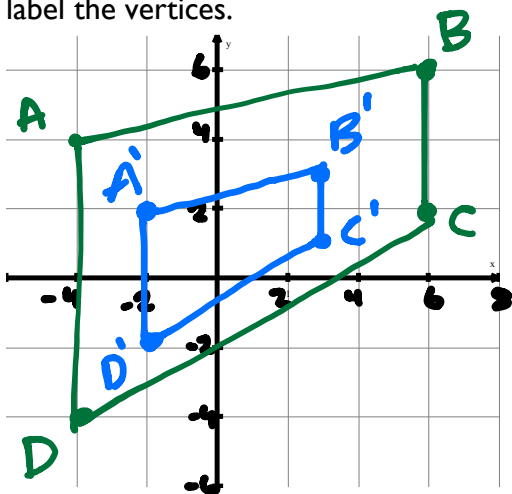
Example 2:

Given the polygon with vertices $A(-4,4), B(6,6), C(6,2), D(-4,-4)$.

a. Determine the new vertices if the triangle is dilated by a scale factor of $\frac{1}{2}$ and centered at $(0,0)$.

$$\begin{aligned} A(-4,4) &\rightarrow A'(-2,2) & C(6,2) &\rightarrow C'(3,1) \\ B(6,6) &\rightarrow B'(3,3) & D(-4,-4) &\rightarrow D'(-2,-2) \end{aligned}$$

b. Draw $ABCD$ and $A'B'C'D'$ on the same plane and label the vertices.

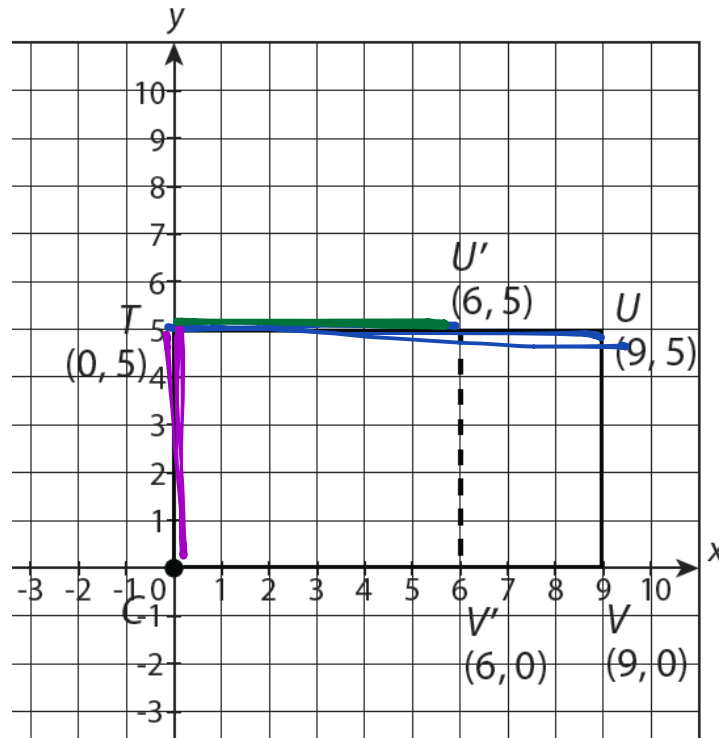


c. Verify that the ratios of corresponding parts are equal to the scale factor.

Properties of Dilations

1. Shape, orientation, and angles are preserved.
2. All sides change by a single scale factor, k .
3. The corresponding preimage and image sides are parallel.
4. The corresponding points of the figure are collinear with the center of dilation.

Example 3: Is the following transformation a dilation? Justify your answer using the properties of dilations.



1. Verify that shape, orientation, and angles have been preserved from the pre-image to the image.

The preimage and image are both rectangles with the same orientation. The angle measures have been preserved since all angles are right angles.

2. Verify that the corresponding sides are parallel.

$\overline{TU} \parallel \overline{T'U'}$
 $\overline{CV} \parallel \overline{C'V'}$
 $\overline{TC} \parallel \overline{T'C'}$

same line

$\overline{UV} \parallel \overline{U'V'}$ → vertical

All corresponding parts are parallel.

3. Verify that the distances of the corresponding sides have changed by a common scale factor, k .

$$\frac{TU'}{TU} = \frac{6}{9} = \frac{2}{3}$$

Horizontal lengths have a scale factor of $\frac{2}{3}$.

$$\frac{TC}{T'C'} = \frac{5}{5} = 1$$

Vertical lengths have a scale factor of 1.

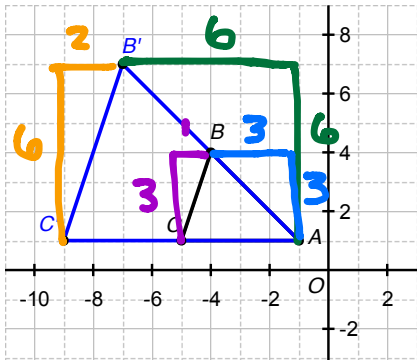
$$\frac{CV'}{CV} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{U'V'}{UV} = \frac{5}{5} = 1$$

4. Draw your conclusion.

This is NOT a dilation because the corresponding sides do not have a common scale factor.

Example 4:



a. Determine the side lengths of each figure.

$$|A'B'| = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$|AB| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$|B'C'| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$$|BC| = \sqrt{9+1} = \sqrt{10}$$

$$|A'C'| = 8$$

$$|AC| = 4$$

b. Write the ratios for corresponding parts between the image and the pre-image.

$$\frac{|A'B'|}{|AB|} = \frac{6\sqrt{2}}{3\sqrt{2}} = \frac{6}{3} = 2$$

$$\frac{|B'C'|}{|BC|} = \frac{2\sqrt{10}}{\sqrt{10}} = 2$$

$$\frac{|A'C'|}{|AC|} = \frac{8}{4} = 2$$

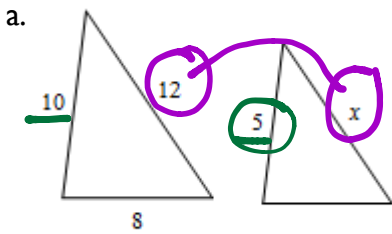
Scale factor = 2

c. Determine if the figures are similar. If yes, write a similarity statement.

They are similar because the corresponding sides have a common scale factor of 2.

$$\triangle ABC \sim \triangle A'B'C'$$

Example 5: The polygons in each pair are similar. State the scale factor and then find the missing side.

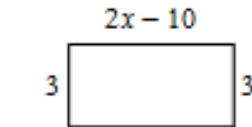
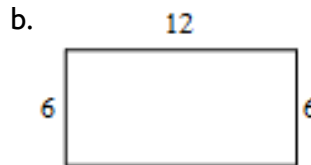


Scale factor: $\frac{5}{10} = \frac{1}{2}$

Scale factor = $\frac{1}{2}$

$$\frac{5}{10} \times \frac{x}{8}$$

$$\frac{60}{10} = \frac{10x}{10} \quad \boxed{x=6}$$



Scale factor = $\frac{1}{2}$

$$\frac{3}{6} \times \frac{2x-10}{12}$$

$$\frac{36}{6} = \frac{6(2x-10)}{6}$$

$$6 = 2x - 10$$

$$\begin{array}{r} +10 \\ 6 = 2x - 10 \\ \hline 16 = 2x \\ \frac{16}{2} = \frac{2x}{2} \end{array} \quad \boxed{x=8}$$