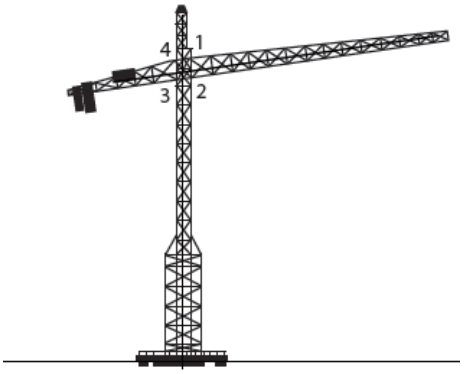
NOTES: SECONDARY 2 HONORS

UNIT 7: Proving Theorems about Lines, Angles and Parallelograms

STARTER:

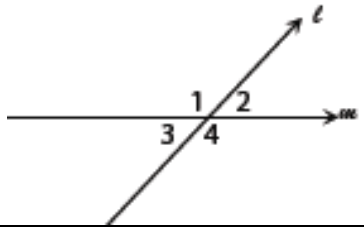
Metalbro is a construction company involved with building a new skyscraper in Dubai. The diagram below is a rough sketch of a crane that Metalbro workers are using to build the skyscraper. The vertical line represents the support tower and the other line represents the boom. The safety reasons, the boom cannot be more than $15°$ beyond the horizon in either direction. A horizontal line forms a $90°$ angle with the support tower.



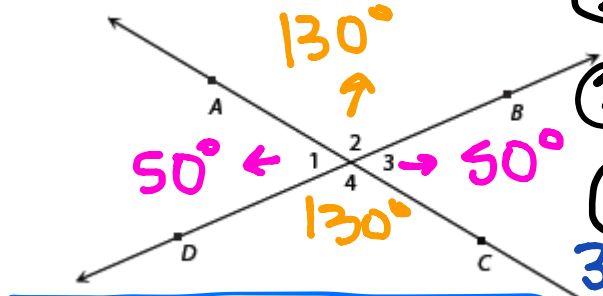
	Based on lower boundary of $\angle 1$	Based on upper boundary of $\angle 1$
$m\angle 1$	75°	105°
$m\angle 2$	105°	75°
$m\angle 3$	75°	105°
$m\angle 4$	105°	75°

RECALL:

- Two angles that are **adjacent** are $\angle 1$ and $\angle 2$.
- Two angles that form a **linear pair** are $\angle 1$ and $\angle 3$.
- Two angles that are **vertical** are $\angle 1$ and $\angle 4$.



Example 1: In the diagram below, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are interesting lines. If $m\angle 1 = 3x + 14$ and $m\angle 2 = 9x + 22$. Find $m\angle 3$ and $m\angle 4$. Justify your steps using postulates and theorems.



$m\angle 3 = 50°$ $m\angle 4 = 130°$

② $m\angle 1 \cong m\angle 3$ because vertical angles are congruent

② $m\angle 2 \cong m\angle 4$ because vertical angles are congruent

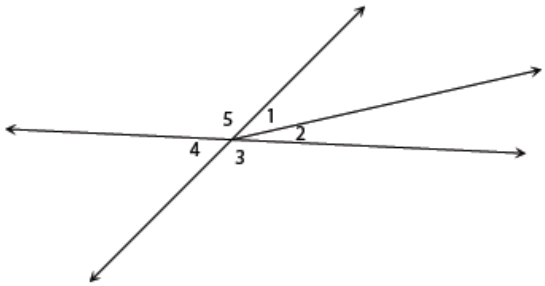
① $m\angle 1 + m\angle 2 = 180$ because they form a linear pair and linear pairs are supplementary.

$3x + 14 + 9x + 22 = 180$

$12x = 144$

$x = 12$

Example 2: If $m\angle 1 = x + 7$, $m\angle 2 = 2(x + 2)$, and $m\angle 4 = 2(x + 13)$ in the diagram below, find $m\angle 4$.



$m\angle 1 + m\angle 2 = m\angle 4$ because vertical angles are congruent.

$x + 7 + 2(x + 2) = 2(x + 13)$

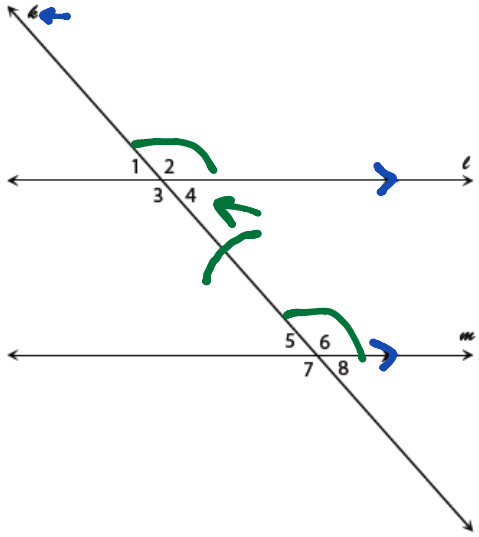
$x + 7 + 2x + 4 = 2x + 26$

$3x + 11 = 2x + 26$

$x = 15$

$m\angle 4 = 2(15 + 13) = 2(28)$

$m\angle 4 = 56°$



The **transversal** is line k

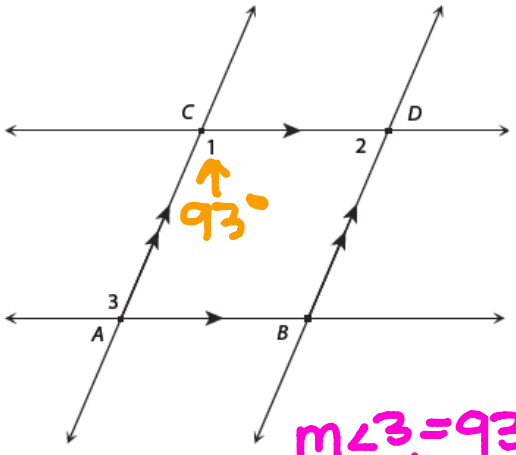
Two **Corresponding Angles** are ∠2 and ∠6. CONGRUENT

Two **Alternate Interior Angles** are ∠4 and ∠5. CONGRUENT

Two **Alternate Exterior Angles** are ∠1 and ∠8. CONGRUENT

Two **Same-Side Interior Angles** are ∠3 and ∠5. Supplementary

Example 3: In the following diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$. If $m\angle 1 = 3(x+15)$, $m\angle 2 = 2x+55$, and $m\angle 3 = 4y+9$, find the measures of the unknown angles and the values of x and y.



m∠1+m∠2=180 because they are same-side interior angles and same-side interior are supplementary.

$3(x+15)+2x+55=180$

$3x+45+2x+55=180$

$5x=80$

$x=16$

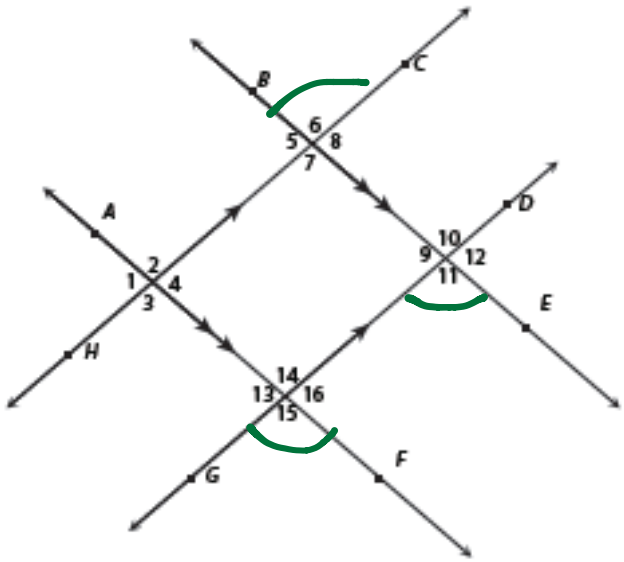
$m\angle 1 = 3(16+15)$

$3(31)$

$=93°$

m∠3=93 because ∠3 is congruent to ∠1 by alternate interior angles.

Example 4: Given two sets of parallel lines in the diagram below, what is the relationship between $\angle 6$ and $\angle 15$? Justify your answer.



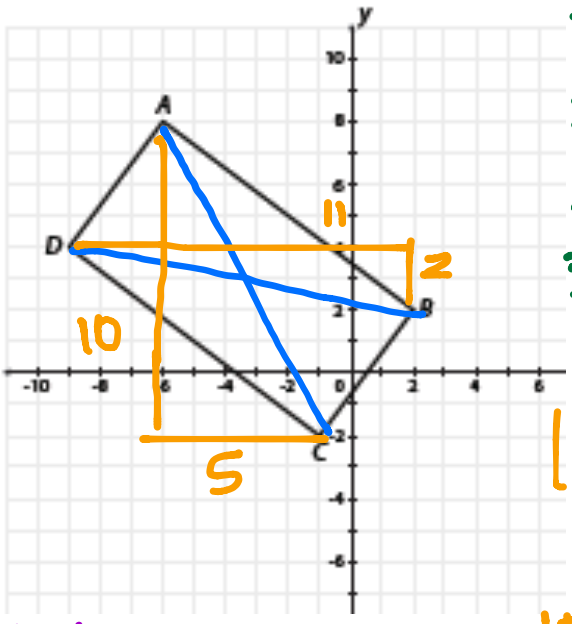
∠6 and ∠15 are congruent because ∠6 and ∠11 are alternate exterior angles and alternate exterior angles are congruent and ∠11 and ∠15 are congruent by corresponding angles.

$\angle 6 \cong \angle 11$ $\angle 11 \cong \angle 15$

$\angle 6 \cong \angle 15$

PROPERTIES OF QUADRILATERALS	
Parallelogram	If a quadrilateral is a parallelogram, • opposite sides are congruent • opposite angles are congruent • consecutive angles are supplementary • the diagonals bisect each other • the diagonal forms two congruent triangles
Rectangle	If a parallelogram is a rectangle, • the angles are all congruent • the diagonals are congruent
Rhombus	If a parallelogram is a rhombus, • all four sides are congruent • the diagonals of a rhombus bisect the opposite pairs of angles • the diagonals are perpendicular
Square	A square has all the properties of a rectangle and a rhombus
Trapezoid	If a quadrilateral is a trapezoid, • exactly one pair of parallel lines A trapezoid is an **isosceles trapezoid** if the nonparallel lines are congruent. • The diagonals of an isosceles trapezoid are congruent.
Kite	If a quadrilateral is a kite, • there are two distinct pairs of congruent sides that are adjacent • the diagonals are perpendicular ** A kite is not a parallelogram **

Example 5: Quadrilateral $ABCD$ has vertices $A(-6,8)$, $B(2,2)$, $C(-1,-2)$, and $D(-9,4)$. Using slope, distance, and/or midpoints, classify $ABCD$ as a rectangle, rhombus, square, trapezoid, isosceles trapezoid, or kite.



Slope

$\overline{AB} = \frac{-6}{8} = \frac{-3}{4}$

$\overline{DC} = \frac{-6}{-8} = \frac{3}{4}$

$\overline{AB} \parallel \overline{DC}$

$\overline{AD} = \frac{4}{3}$

$\overline{BC} = \frac{4}{3}$

$\overline{AD} \parallel \overline{BC}$

Distance

$|\overline{AC}| = 5^2 + 10^2 = x^2$

$25 + 100 = x^2$

$125 = x^2$

$x = 5\sqrt{5}$

$\overline{AC} \cong \overline{BD}$

$|BD| = 11^2 + 2^2 = x^2$

$121 + 4 = x^2$

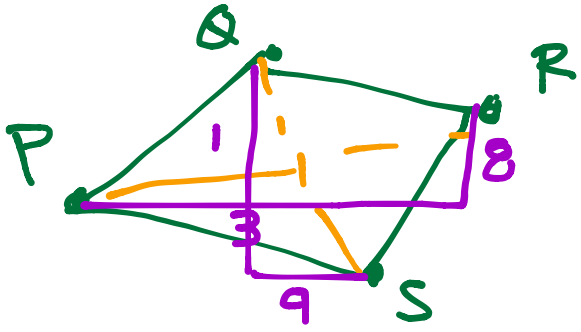
$125 = x^2$

$x = 5\sqrt{5}$

Quadrilateral ABCD is a rectangle since all four sides are parallel and the diagonals are congruent.

Use what you know about the diagonals of rectangles, rhombuses, squares, kites, and trapezoids to classify each given quadrilaterals.

Example 6: Quadrilateral $PQRS$ has vertices $P(1,5)$, $Q(5,2)$, $R(4,-3)$, and $S(-4,3)$.



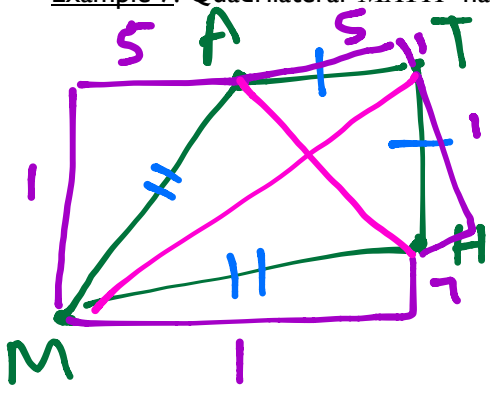
$|\overline{PR}| = 3^2+8^2=x^2$
$9+64=x^2$
$73=x^2$
$x=\sqrt{73}$

$|\overline{QS}| = 1^2+9^2=x^2$
$1+81=x^2$
$82=x^2$
$x=\sqrt{82}$

Slope
$\overline{PS}=\frac{2}{5}$
$\overline{QR}=\frac{5}{1}=5$
$\overline{PQ}=\frac{-3}{4}$
$\overline{RS}=\frac{-6}{8}=\frac{-3}{4}$
$\overline{PQ} \parallel \overline{RS}$

Quadrilateral PQRS is a trapezoid since there is exactly one pair of parallel lines and the diagonals are not congruent.

Example 7: Quadrilateral $MATH$ has vertices $M(0,3)$, $A(5,2)$, $T(6,-3)$, and $H(-1,-4)$.



Slopes
$\overline{AT}=\frac{-5}{1}=-5$
$\overline{MH}=\frac{-7}{-1}=+7$
$\overline{MA}=\frac{1}{-5}$
$\overline{TH}=\frac{-1}{-7}=\frac{1}{7}$
$\overline{MT}=\frac{-6}{6}=-1$
$\overline{AH}=\frac{-6}{-6}=1$

Distances
$\overline{MA}=5^2+1^2=x^2$
$25+1=x^2$
$26=x^2$
$x=\sqrt{26}$

$\overline{AT}=5^2+1^2=x^2$
$x=\sqrt{26}$

$\overline{TH}=7^2+1^2=x^2$
$49+1=x^2$
$50=x^2$
$x=5\sqrt{2}$

$\overline{MH}=7^2+1^2=x^2$
$x=5\sqrt{2}$

Quadrilateral MATH is a kite two distinct congruent sides that are adjacent and the diagonals are perendicular.