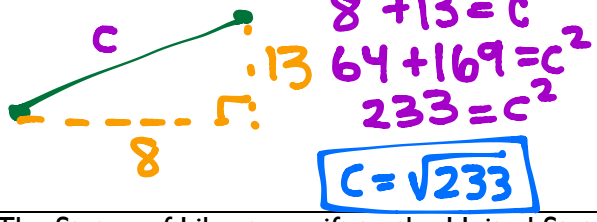
NOTES: SECONDARY 2 HONORS

# UNIT 7: Proving Similarity

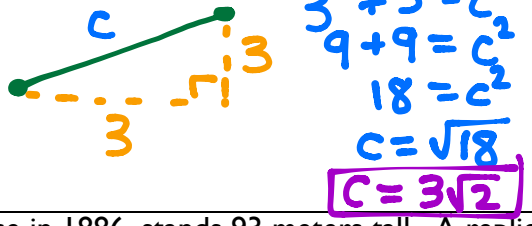
STARTER

1. Find the distance between the two points:

a. $(3, 7)$ and $(-5, -6)$




b. $(-1, 4)$ and $(-4, 1)$




2. The Statue of Liberty, a gift to the United States from France in 1886, stands 93 meters tall. A replica of the famous statue is 7.75 meters.

a. What is the scale factor comparing the height of the replica to the actual height of the Statue of Liberty?

$\frac{7.75}{93} = \frac{1}{12}$

b. The height of the base of the Statue of Liberty is approximately 46.5 meters. What is the height of the base of the Statue of Liberty Replica?

$46.5\left(\frac{1}{12}\right) = 3.87\text{ m}$

**Methods for Proving Triangle Similarity:**

➔ **Angle-Angle (AA) Similarity Statement**: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

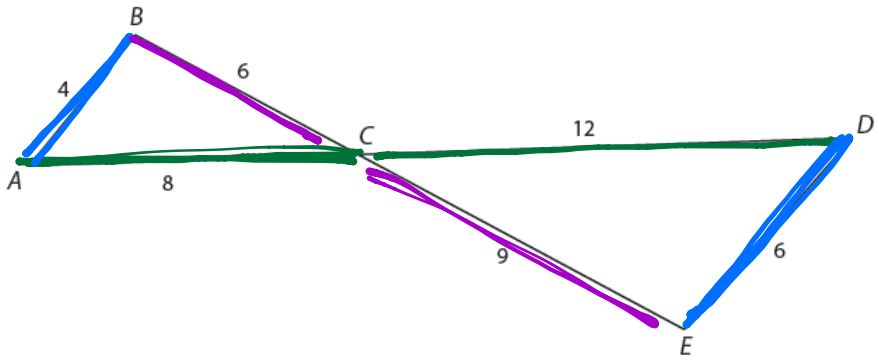
➔ **Side-Angle-Side (SAS) Similarity Statement**: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent, then the triangles are similar.

➔ **Side-Side-Side (SSS) Similarity Statement**: If the measure of the three sides of two triangles are proportional, then the triangles are similar.

Legitimate **proofs** include the following:

- A statement of what is to be proven
- A list of the given information
- A diagram including the given information (if possible)
- Step-by-step statements to support logical reasoning

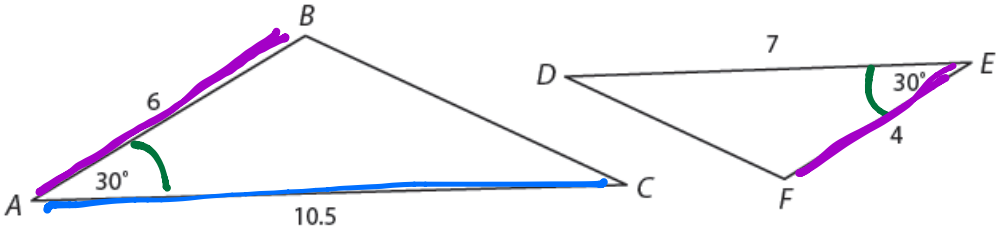
**Example 1:** Prove that $\Delta ABC \sim \Delta DEC$.




$\Delta ABC \sim \Delta DEC$ by SSS since the three sides have a common scale factor 3/2 making them Proportional

$\frac{|CD|}{|AC|} = \frac{12}{8} = \frac{3}{2}$ $\frac{|DE|}{|AB|} = \frac{6}{4} = \frac{3}{2}$ $\frac{|CE|}{|BC|} = \frac{9}{6} = \frac{3}{2}$

**Example 2:** Determine whether the following triangles are similar. Explain your reasoning logically.



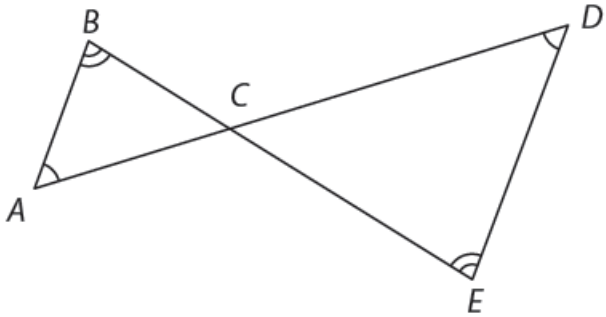

$\angle A = \angle E$

$\frac{|AB|}{|EF|} = \frac{6}{4} = \frac{3}{2}$

$\frac{|AC|}{|DE|} = \frac{10.5}{7} = \frac{3}{2}$

$\Delta ABC \sim \Delta EFD$ by SAS since two corresponding sides have a common scale factor of $\frac{3}{2}$ making them proportional and the included angle is congruent.

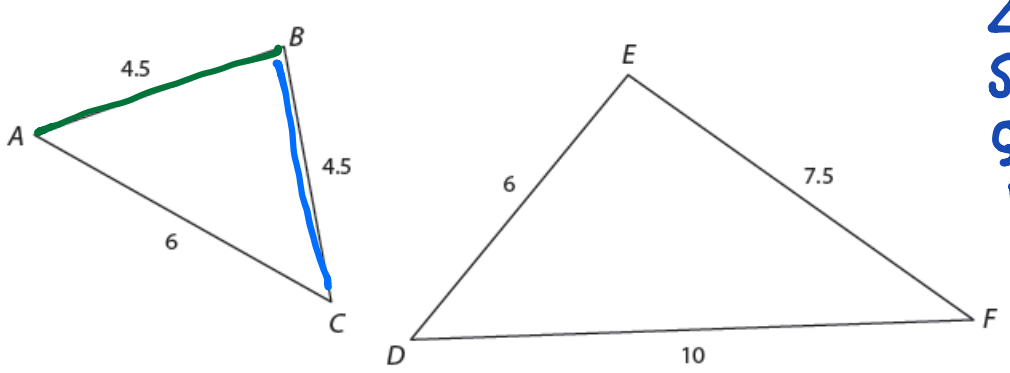
**Example 3:** Determine if the following diagram contains similar triangles. Explain your reasoning logically.




$\angle A \cong \angle D$ $\angle B \cong \angle E$

$\Delta ABC \sim \Delta DEC$ by AA since two corresponding angles are congruent.

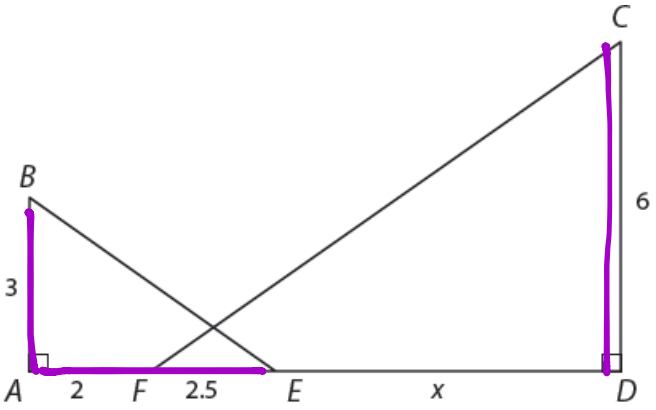
**Example 4:** Determine whether the triangles are similar. Explain your reasoning.




$\triangle ABC \not\sim \triangle DEF$ since the three sides are not proportional.

$\frac{|AB|}{|DE|} = \frac{4.5}{6} = \frac{3}{4}$ $\frac{|BC|}{|EF|} = \frac{4.5}{7.5} = \frac{9}{15} = \frac{3}{5}$

**Example 5:** Identify the similar triangles and then find the value of x.




$\triangle ABE \sim \triangle DCF$

$\frac{3}{6} \times \frac{4.5}{x+2.5}$

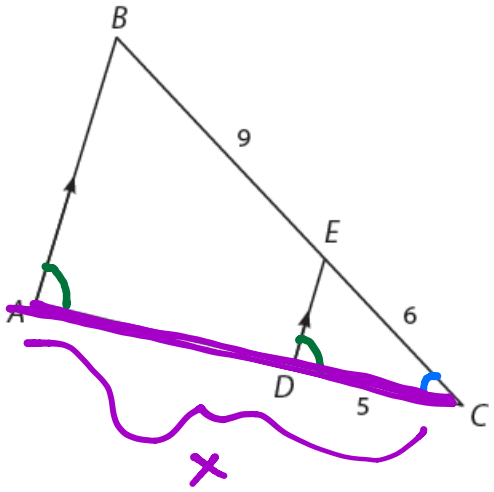
$27 = 3(x+2.5)$

$27 = 3x + 7.5$

$19.5 = 3x$

$x = 6.5 = \frac{13}{2}$

**Example 6:** Identify the similar triangles and prove their similarity. Then, find the length of $\overline{CA}$.




$\angle A \cong \angle D$ by corresponding angles

$\angle C \cong \angle C$ by reflexive prop.

$\triangle ABC \sim \triangle DEC$ by AA since two corresponding angles are congruent.

$\frac{x}{5} \times \frac{15}{6}$ $6x = 75$ $\frac{6x}{6} = \frac{75}{6}$

$\overline{CA} = 12.5 = \frac{25}{2}$

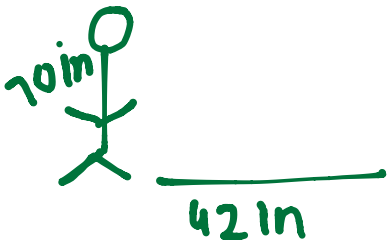
**Triangle Proportionality Theorem:** If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the parallel line divides these two sides proportionally.

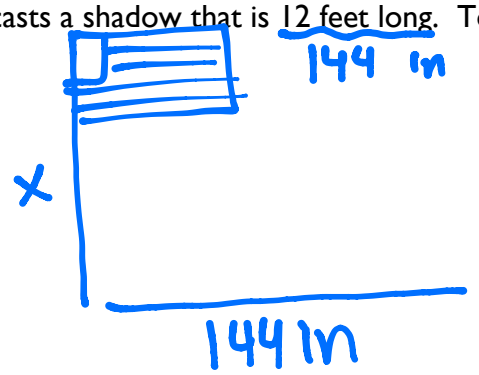
$\frac{x}{9} = \frac{5}{6}$

70 in

42 in

**Example 7:** Suppose a person who is 5 feet 10 inches tall casts a shadow that is 3 feet 6 inches long. At the same time of day, a flagpole casts a shadow that is 12 feet long. To the nearest foot, how tall is the flagpole?



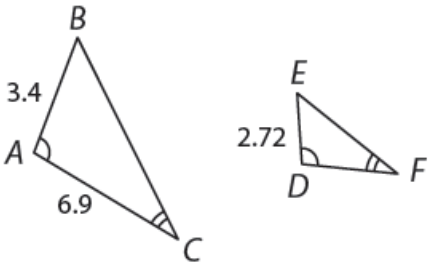

$\frac{70}{x} \times \frac{42}{144}$

$10080 = 42x$

$x = 240$ in

$x = 20$ ft

**Example 8:** Explain why $\Delta ABC \sim \Delta DEF$, and then find the length of $\overline{DF}$.



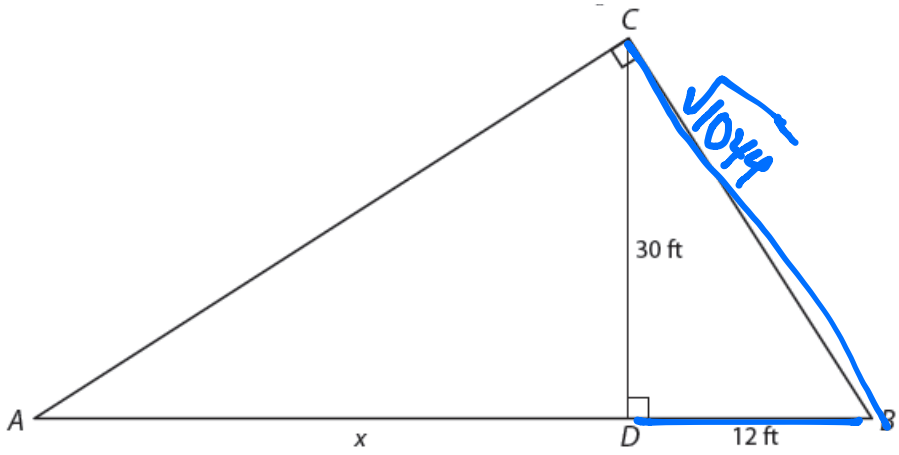

$\Delta ABC \sim \Delta DEF$ by AA since two corresponding angles are congruent.

$\frac{3.4}{2.72} \times \frac{6.9}{x}$

$3.4x = 18.768$

$x = 5.52$

**Example 9:** To find the distance across a pond, Rita climbs a 30-foot observation tower on the shore of the pond and locates points A and B so that $\overline{AC}$ is perpendicular to $\overline{CB}$. She finds the measure of $\overline{DB}$ to be 12 feet. What is the measure of $\overline{AD}$, the distance across the pond?




$\Delta ABC \sim \Delta CBD$ by AA since two corresponding angles are congruent

$\frac{12}{30} = \frac{\sqrt{1044}}{x+12}$

$30\sqrt{1044} = 12x + 144$

$825.33 = 12x$

$x = 68.78$