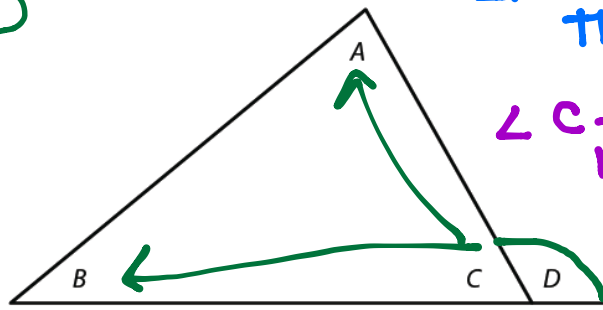


NOTES: SECONDARY 2 HONORS
UNIT 7: Proving Theorems About Triangles

Triangle Sum Theorem: The sum of the interior angles of a triangle is 180°

Exterior Angle Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

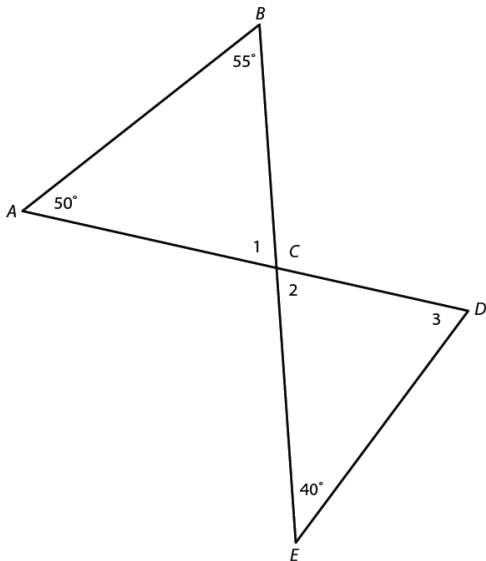
$$\angle A + \angle B = \angle D$$



$\angle A + \angle B + \angle C = 180$ by the Triangle Sum Theorem
 $\angle C + \angle D = 180$ because linear pairs are supplementary.

$$\begin{aligned} \angle A + \angle B + \cancel{\angle C} &= \angle C + \angle D \\ &\quad -\cancel{\angle C} \quad -\cancel{\angle C} \\ \angle A + \angle B &= \angle D \quad \checkmark \end{aligned}$$

Example 1: Find the missing angle measures. Justify your steps.



$\angle A + \angle B + \angle 1 = 180$ by the Triangle Sum Theorem.

$$50 + 55 + \angle 1 = 180$$

$$105 + \angle 1 = 180$$

$$\boxed{\angle 1 = 75^\circ}$$

$\angle 1 \cong \angle 2$ because vertical angles are congruent.

$$\boxed{75^\circ = \angle 2}$$

$\angle E + \angle 2 + \angle 3 = 180$ by the Triangle Sum Theorem

$$40 + 75 + \angle 3 = 180$$

Example 2: Find the missing angle measures. Justify your steps.

$$115 + \angle 3 = 180$$

$$\boxed{\angle 3 = 65^\circ}$$

$3x - 10 + 4x + 60 = 155$ by the Exterior Angle Theorem

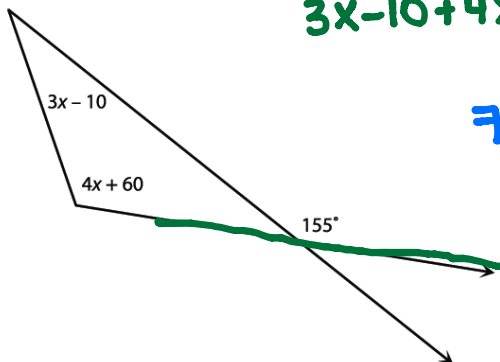
$$7x + 50 = 155$$

$$7x = 105$$

$$x = 15$$

$$3(15) - 10 = \boxed{35^\circ}$$

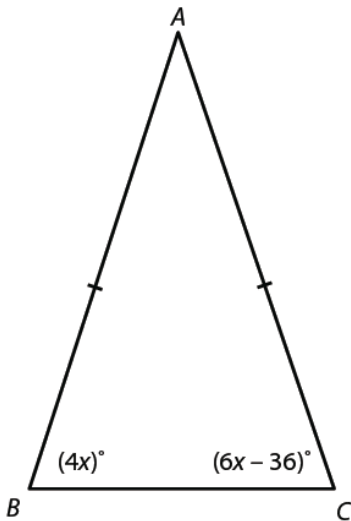
$$4(15) + 60 = \boxed{120^\circ}$$



Isosceles Triangle Theorem: If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.

→ The converse is also true and is called the **Converse of the Isosceles Triangle Theorem**

Example 3: Find the measure of each angle in $\triangle ABC$. Justify your steps.



$\triangle ABC$ is an isosceles triangle since two sides are congruent.

$\angle B \cong \angle C$ by the Isosceles Triangle Theorem.

$$\begin{aligned} 4x &= 6x - 36 \\ 0 &= 2x - 36 \\ 36 &= 2x \\ \underline{x} &= \underline{18} \end{aligned}$$

$$\begin{aligned} \angle B &= 4(18) = \\ \angle B &= 72^\circ \end{aligned}$$

$$\begin{aligned} \angle C &= 6(18) - 36 \\ \angle C &= 72^\circ \end{aligned}$$

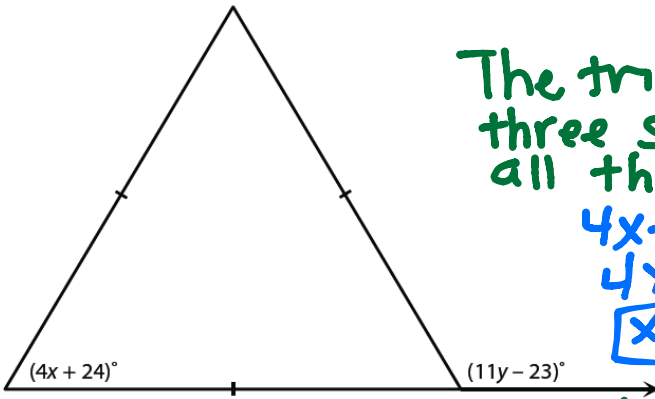
$\angle A + \angle B + \angle C = 180$ by the Triangle Sum Theorem.

$$\angle A + 72 + 72 = 180$$

$$\angle A + 144 = 180$$

$$\angle A = 36^\circ$$

Example 4: Find the values of x and y . Justify your steps.



The triangle is equilateral since all three sides are congruent. Therefore, all three angles are congruent at 60° .

$$4x + 24 = 60$$

$$4x = 36$$

$$\underline{x} = \underline{9}$$

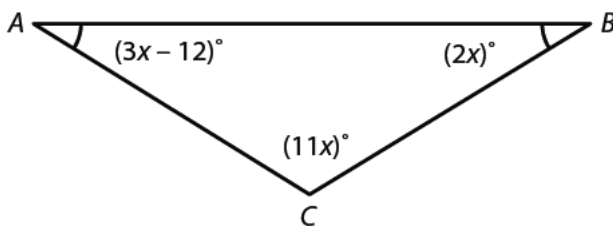
$60 + 60 = 11y - 23$ by the Exterior Angle Theorem.

$$120 = 11y - 23$$

$$143 = 11y$$

$$\underline{y} = \underline{13}$$

Example 5: Find $m\angle A$, $m\angle B$, and $m\angle C$. Justify your steps.



$\angle A \cong \angle B$ (given)

$$3x - 12 = 2x$$

$$-12 = -x$$

$$\underline{x} = \underline{12}$$

$$\angle A = 3(12) - 12$$

$$\angle A = 24^\circ$$

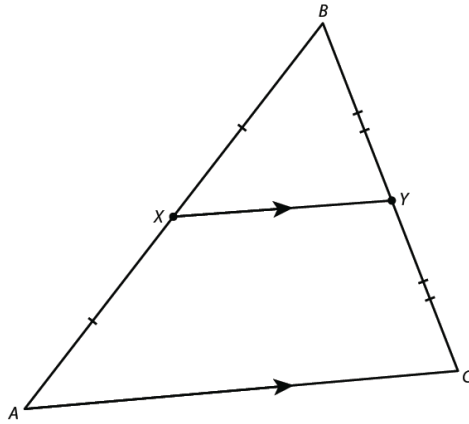
$$\angle B = 2(12)$$

$$\angle B = 24^\circ$$

$$\angle C = 11(12)$$

$$\angle C = 132^\circ$$

Triangle Midsegment Theorem: A midsegment (the line joining the midpoints of two sides of a triangle) of a triangle is parallel to the third side and is half the length of the third side.



Example 6: Find the lengths of BC and YZ and the measure of $\angle AXZ$. Justify your steps.

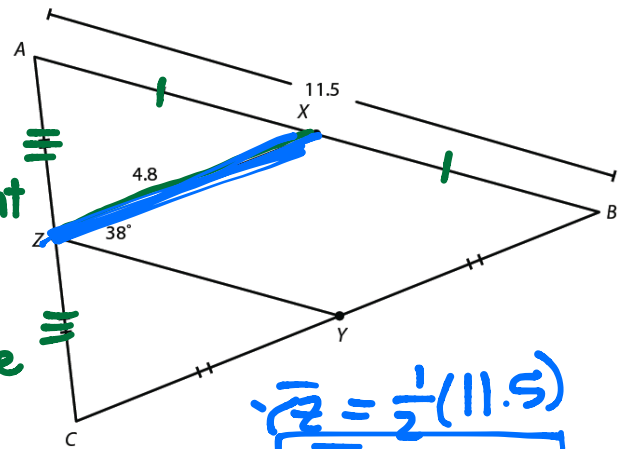
\overline{XZ} is a midsegment of $\triangle ABC$ since \overline{XZ} bisects \overline{AC} and \overline{AB} .

$\overline{XZ} = \frac{1}{2} \overline{BC}$ by the Triangle Midsegment Theorem

$4.8 = \frac{1}{2} \overline{BC}$ $\overline{BC} = 9.6$

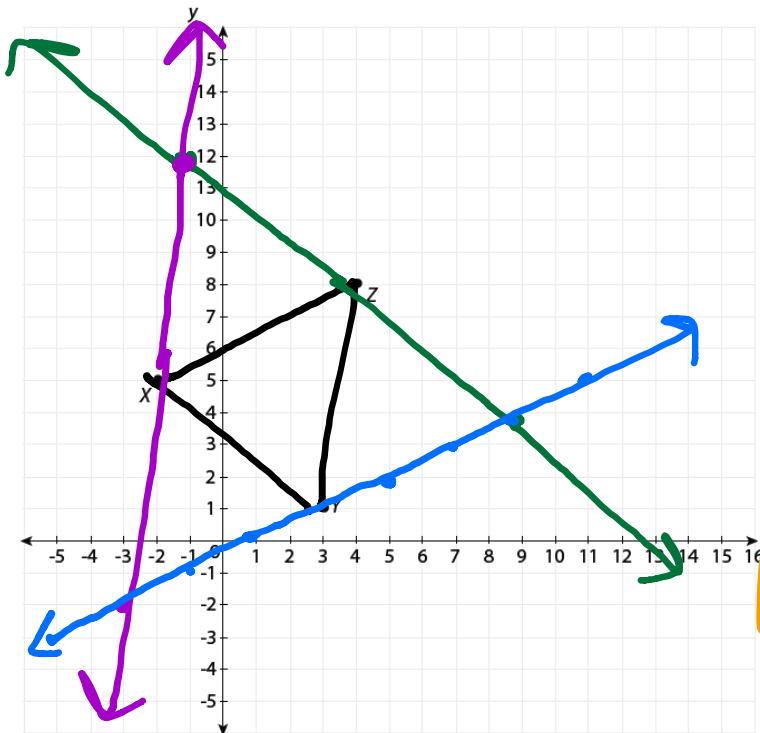
\overline{YZ} is a midsegment of $\triangle ABC$ since it bisects \overline{AC} and \overline{BC} .

$\overline{YZ} = \frac{1}{2} \overline{AB}$ by the triangle Midsegment Theorem.



$\overline{YZ} = \frac{1}{2}(11.5)$
 $\overline{YZ} = 5.75$

Example 7: The midpoints of a triangle's sides are X (-2, 5), Y (3, 1) and Z (4, 8). Find the coordinates of the vertices of the triangle.



Slope
 $\overline{XZ} = -\frac{4}{9}$

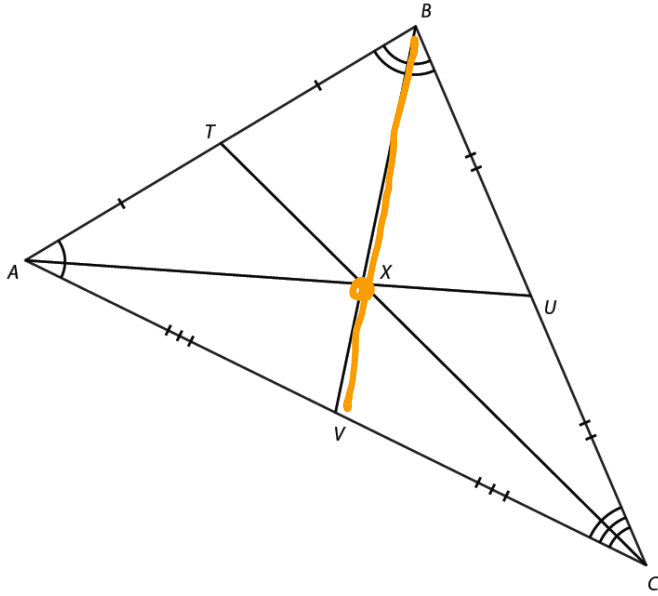
$\overline{YZ} = 7$

$\overline{XY} = \frac{1}{2}$

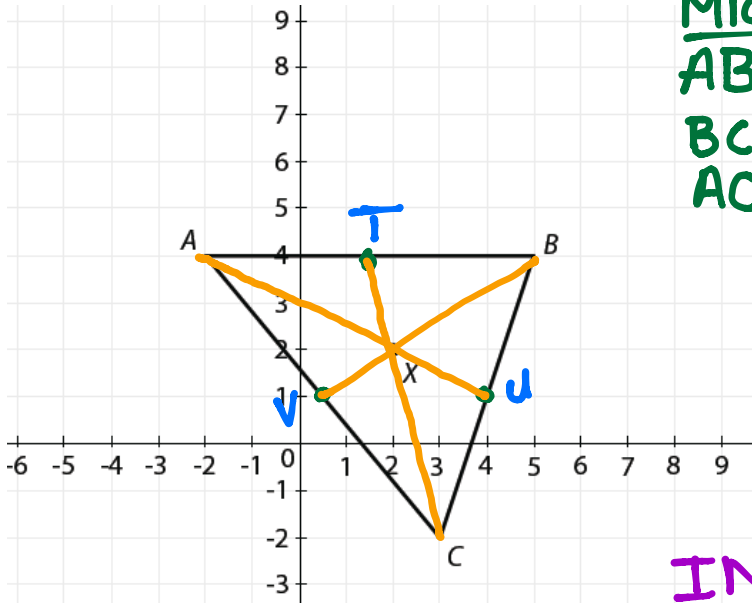
vertices:
 $(-1, 12); (9, 4); (-3, -2)$

The Centroid Theorem: The centroid of a triangle is $\frac{2}{3}$ the distance from each vertex to the midpoint of the opposite side.

- The **centroid** is the point where all three medians of a triangle meet.
- A **median** is a segment that joins the vertex of triangle to the midpoint of the opposite side.



Example 8: Triangle ABC has vertices A (-2, 4), B (5, 4) and C (3, -2). Find the equation of each median of ΔABC to verify that (2, 2) is the centroid of ΔABC .



Midpoints
 $AB = (1.5, 4)$
 $BC = (4, 1)$
 $AC = (1, 1)$

Equation
 $\overline{AU} \Rightarrow m = \frac{1}{2}$
 $(-2, 4)$
 $4 = \frac{1}{2}(-2) + b$
 $4 = 1 + b$
 $b = 3$
 $\overline{AU} = y = \frac{1}{2}x + 3$
 $\overline{BV} = y = \frac{2}{3}x + \frac{2}{3}$
 $\overline{CT} = y = -4x + 10$

Intersect: (2, 2)
 $\overline{AU} = y = \frac{1}{2}(2) + 3$
 $y = -1 + 3$
 $y = 2 \checkmark$
 $\overline{BV} = y = \frac{2}{3}(2) + \frac{2}{3}$
 $y = \frac{4}{3} + \frac{2}{3}$
 $y = 2 \checkmark$

The point (2, 2) is the centroid of ΔABC because it's the intersection of all three medians.

$\overline{CT} = -4(2) + 10 = -8 + 10 \Rightarrow y = 2 \checkmark$

Classwork Question: Determine whether ΔABC with vertices A (-4, 5), B (-1, -4) and C (5, 2) is an isosceles triangle. If it is isosceles, name a pair of congruent angles and a pair of congruent sides.