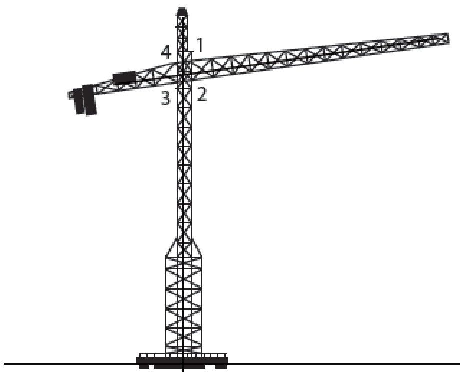
NOTES: SECONDARY 2 HONORS
UNIT 7: Proving Theorems about Lines, Angles and Parallelograms

STARTER:

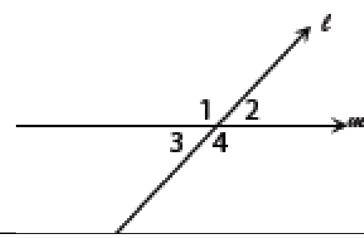
Metalbro is a construction company involved with building a new skyscraper in Dubai. The diagram below is a rough sketch of a crane that Metalbro workers are using to build the skyscraper. The vertical line represents the support tower and the other line represents the boom. The safety reasons, the boom cannot be more than $15°$ beyond the horizon in either direction. A horizontal line forms a $90°$ angle with the support tower.



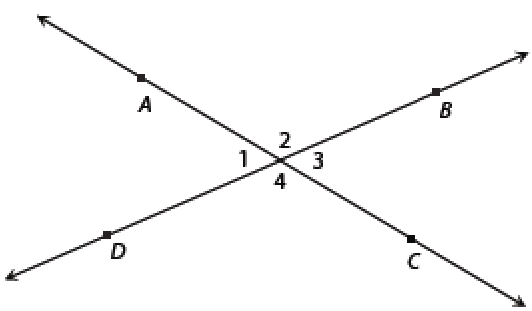
	Based on lower boundary of $\angle 1$	Based on upper boundary of $\angle 1$
$m\angle 1$		
$m\angle 2$		
$m\angle 3$		
$m\angle 4$		

RECALL:

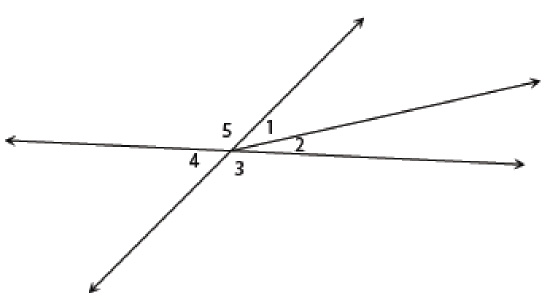
- Two angles that are **adjacent** are ________ and ________.
- Two angles that form a **linear pair** are ________ and ________.
- Two angles that are **vertical** are ________ and ________.

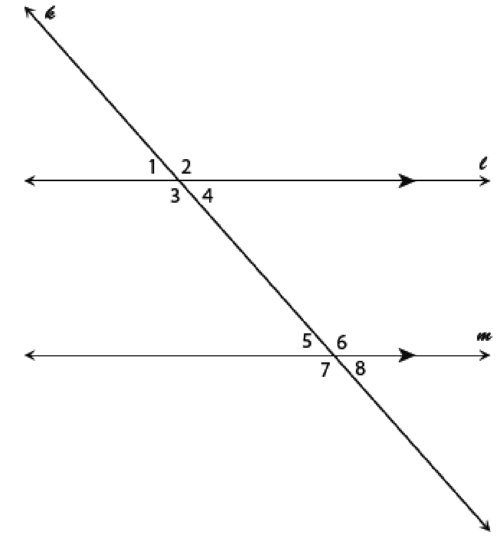


Example 1: In the diagram below, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are interesting lines. If $m\angle 1 = 3x + 14$ and $m\angle 2 = 9x + 22$. Find $m\angle 3$ and $m\angle 4$. Justify your steps using postulates and theorems.

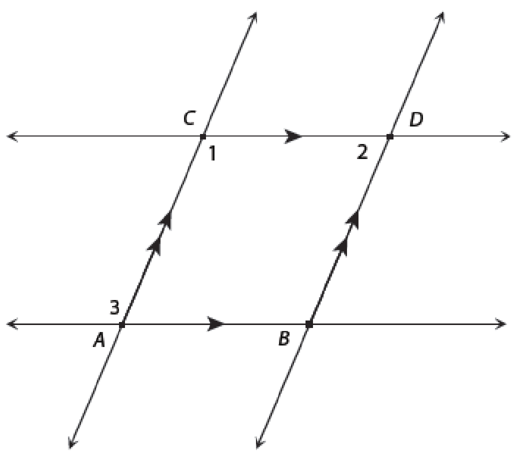


Example 2: If $m\angle 1 = x + 7$, $m\angle 2 = 2(x+2)$, and $m\angle 4 = 2(x+13)$ in the diagram below, find $m\angle 4$.

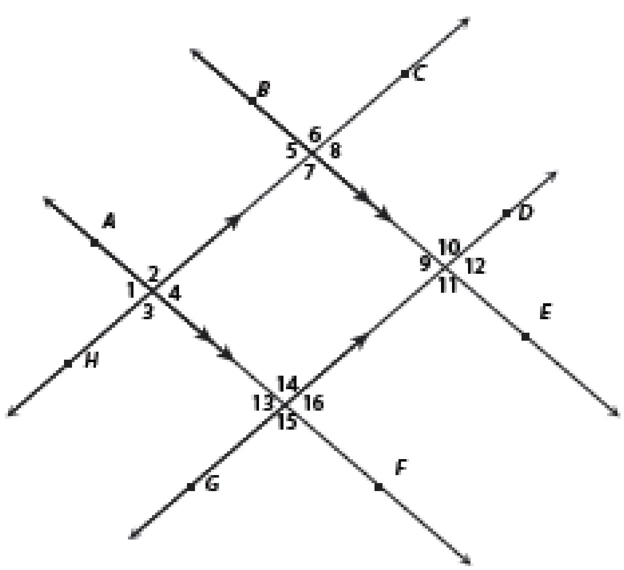


	The **transversal** is line __________
	Two **Corresponding Angles** are ______ and _______.
	Two **Alternate Interior Angles** are ______ and _______.
	Two **Alternate Exterior Angles** are ______ and _______.
	Two **Same-Side Interior Angles** are ______ and _______.

Example 3: In the following diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$. If $m\angle 1 = 3(x+15)$, $m\angle 2 = 2x+55$, and $m\angle 3 = 4y+9$, find the measures of the unknown angles and the values of x and y.

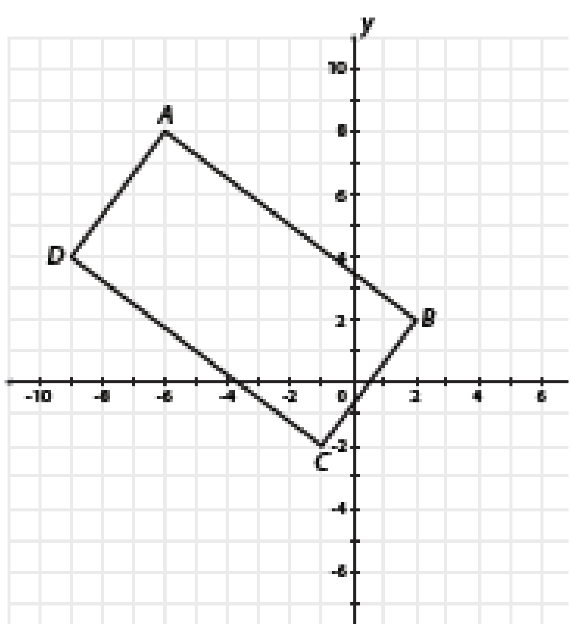


Example 4: Given two sets of parallel lines in the diagram below, what is the relationship between $\angle 6$ and $\angle 15$? Justify your answer.



PROPERTIES OF QUADRILATERALS	
Parallelogram	If a quadrilateral is a parallelogram, • opposite sides are congruent • opposite angles are congruent • consecutive angles are supplementary • the diagonals bisect each other • the diagonal forms two congruent triangles
Rectangle	If a parallelogram is a rectangle, • the angles are all congruent • the diagonals are congruent
Rhombus	If a parallelogram is a rhombus, • all four sides are congruent • the diagonals of a rhombus bisect the opposite pairs of angles • the diagonals are perpendicular
Square	A square has all the properties of a rectangle and a rhombus
Trapezoid	If a quadrilateral is a trapezoid, • exactly one pair of parallel lines A trapezoid is an **isosceles trapezoid** if the nonparallel lines are congruent. • The diagonals of an isosceles trapezoid are congruent.
Kite	If a quadrilateral is a kite, • there are two distinct pairs of congruent sides that are adjacent • the diagonals are perpendicular ** A kite is not a parallelogram **

Example 5: Quadrilateral $ABCD$ has vertices $A(-6,8)$, $B(2,2)$, $C(-1,-2)$, and $D(-9,4)$. Using slope, distance, and/or midpoints, classify $ABCD$ as a rectangle, rhombus, square, trapezoid, isosceles trapezoid, or kite.



Use what you know about the diagonals of rectangles, rhombuses, squares, kites, and trapezoids to classify each given quadrilaterals.

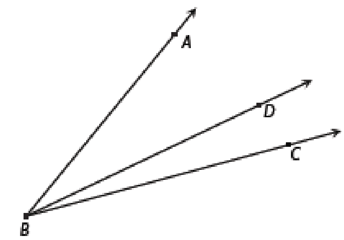
Example 6: Quadrilateral $PQRS$ has vertices $P(1,5)$, $Q(5,2)$, $R(4,-3)$, and $S(-4,3)$.

Example 7: Quadrilateral $MATH$ has vertices $M(0,3)$, $A(5,2)$, $T(6,-3)$, and $H(-1,-4)$.

POSTULATES AND THEOREMS

ANGLE ADDITION POSTULATE

- If *D* is in the interior of $\angle ABC$, then $m\angle ABC + m\angle DBC = m\angle ABC$.
- If $m\angle ABC + m\angle DBC = m\angle ABC$, then *D* is in the interior of $\angle ABC$.



CONGRUENCE OF ANGLES

Reflexive Property:
$\angle 1 \cong \angle 1$

Symmetric Property:
If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$

Transitive Property:
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$

PERPENDICULAR BISECTOR THEOREM

- If $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AC}$, then $DA = DC$.
- If $DA = DC$, then $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AC}$.