NOTES: SECONDARY 2 HONORS

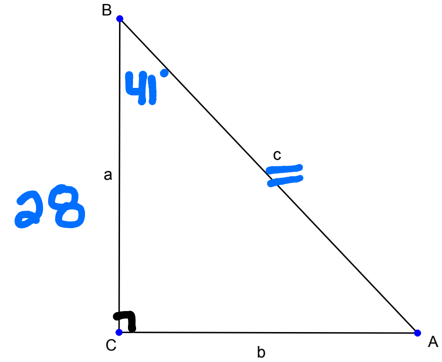
UNIT 9: Applying Trigonometric Ratios

Trigonometry can be used to find:

- unknown side lengths of a triangle
- unknown angles in a triangle

➔ Find a missing side length when an acute angle and one side are known:

Example 1: Use the triangle shown to help you find the missing measure.



a. If $\angle A = 20^\circ$ and c=32, find a.

$32 \cdot \sin 20 = \frac{a}{32} \cdot 32$

$a = 32 \cdot \sin 20$ $\boxed{a \approx 10.945}$

b. If $\angle A = 49^\circ$ and a=17, find b.

$b \cdot \tan 49 = \frac{17}{\cancel{b}} \cdot \cancel{b}$ $\frac{b \cancel{\tan 49}}{\cancel{\tan 49}} = \frac{17}{\tan 49}$

$b = \frac{17}{\tan 49}$ $\boxed{b \approx 14.778}$

c. If $\angle A = 27.3^\circ$ and a=7, find c.

$c \cdot \sin 27.3 = \frac{7}{c} \cdot c$ $\frac{c \cancel{\sin 27.3}}{\cancel{\sin 27.3}} = \frac{7}{\sin 27.3}$

$c = \frac{7}{\sin 27.3}$ $\boxed{c \approx 15.262}$

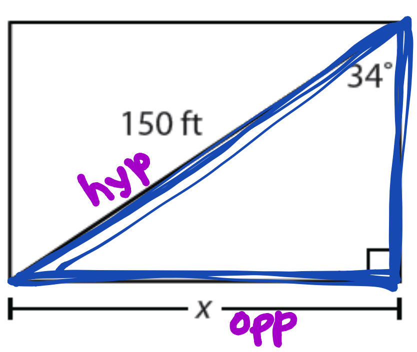
d. If $\angle A = 63.4^\circ$ and a=19.2, find b.

e. If $\angle B = 41^\circ$ and a=28, find c.

$c \cdot \cos 41 = \frac{28}{c} \cdot c$ $\frac{c \cdot \cancel{\cos 41}}{\cancel{\cos 41}} = \frac{28}{\cos 41}$

$c = \frac{28}{\cos 41}$ $\boxed{c \approx 37.100}$

Example 2: Leo is building a concrete pathway 150 feet long across a rectangular courtyard, as shown below. What is the length of the courtyard, x, to the nearest thousandth?



$150 \cdot \sin 34 = \frac{x}{150} \cdot 150$

$x = 150 \sin 34$

$\boxed{x \approx 83.879 \text{ ft}}$

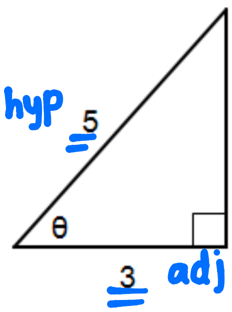
➔ Find a missing angle when two sides are known (in a right triangle):

In order to find missing angles, **inverse trigonometric functions** must be used.

Equation	Inverse Equation	Meaning
$\sin(\theta) = \frac{x}{y}$	$\sin^{-1}\left(\frac{x}{y}\right) = \theta$	The inverse, or **arcsine** of $\frac{x}{y}$ is equal to the angle θ.
$\cos(\theta) = \frac{x}{y}$	$\cos^{-1}\left(\frac{x}{y}\right) = \theta$	The inverse, or **arccosine** of $\frac{x}{y}$ is equal to the angle θ.
$\tan(\theta) = \frac{x}{y}$	$\tan^{-1}\left(\frac{x}{y}\right) = \theta$	The inverse, or **arctangent** of $\frac{x}{y}$ is equal to the angle θ.

Example 3: Use the side lengths to find the angle θ.

a.



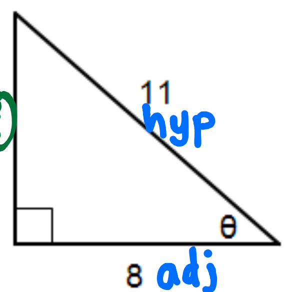
$\cos\theta = \frac{3}{5}$

$\cancel{\cos^{-1}}(\cancel{\cos}\theta) = \cos^{-1}\left(\frac{3}{5}\right)$

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

$\boxed{\theta = 53.130°}$

b.

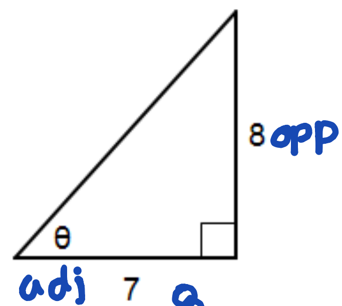


$\cos\theta = \frac{8}{11}$

$\theta = \cos^{-1}\left(\frac{8}{11}\right)$

$\boxed{\theta = 43.342°}$

c.

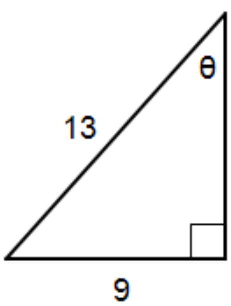


$\tan\theta = \frac{8}{7}$

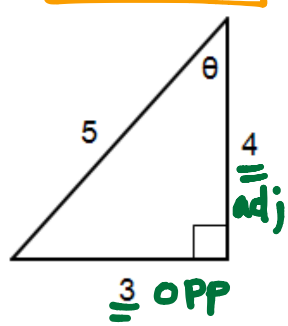
$\theta = \tan^{-1}\left(\frac{8}{7}\right)$

$\boxed{\theta = 48.814°}$

d.



e.

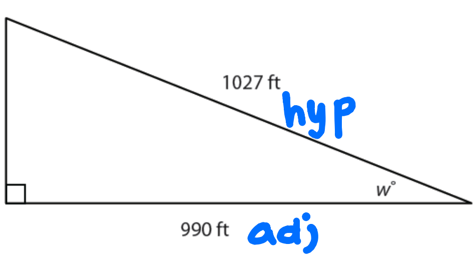


$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\boxed{\theta = 36.870°}$

Example 4: A trucker drives 1,027 feet up a hill that has a constant slope. When the trucker reaches the top of the hill, he has traveled a horizontal distance of 990 feet. At what angle did the trucker drive to reach the top? Round to the nearest degree.



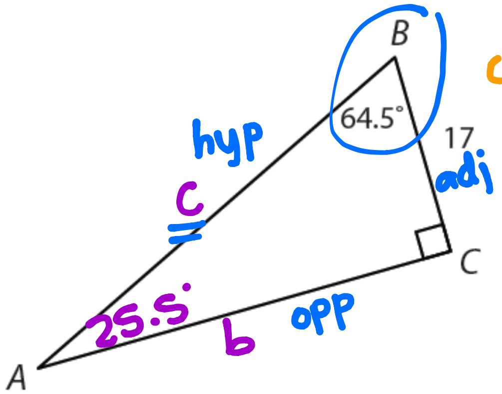
$\cos w° = \frac{990}{1027}$

$w° = \cos^{-1}\left(\frac{990}{1027}\right)$

$\boxed{w° = 15°}$

Example 5: Solve (in other words… find the missing sides and angles) in the triangle below.
Round sides to the nearest thousandth.

$\boxed{\angle A = 25.5^\circ}$



$c \cdot \cos 64.5 = \frac{17}{c} \cdot c$

$\frac{c\cos 64.5}{\cos 64.5} = \frac{17}{\cos 64.5}$

$c = \frac{17}{\cos 64.5}$

$\boxed{c \approx 39.488}$

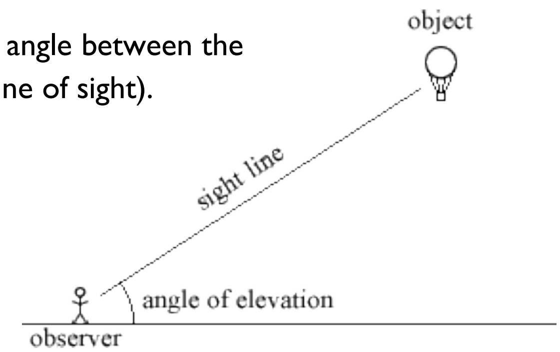
$17 \cdot \tan 64.5 = \frac{b}{17} \cdot 17$

$b = 17 \tan 64.5$

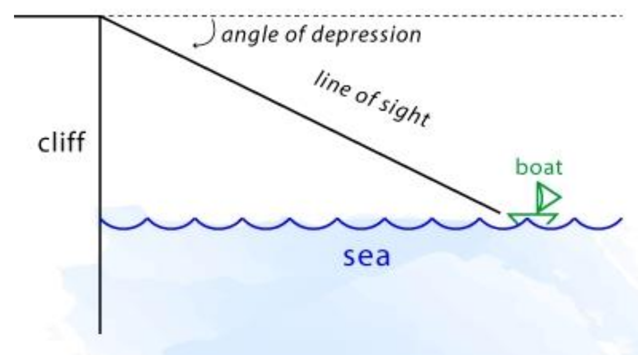
$\boxed{b \approx 35.641}$

Common Applications of Trigonometry:

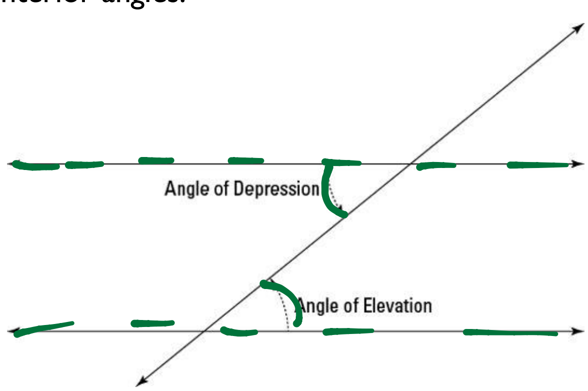
The **angle of elevation** of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).



If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the **angle of depression**.

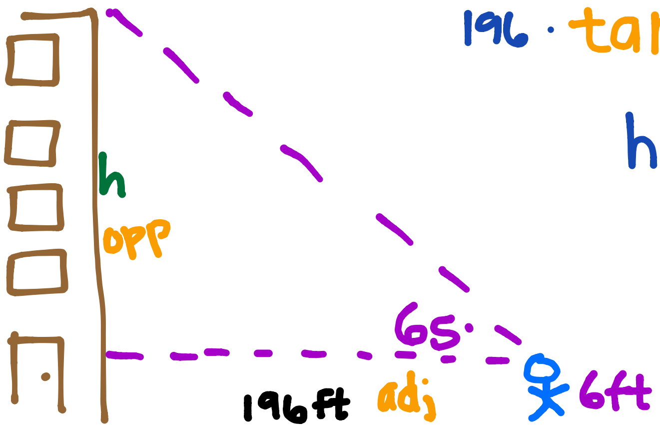


The angle of elevation (of the person looking up) equals the angle of depression (of the person looking down) because they form alternate interior angles.



Alternate Interior Angles.

Example 6: You are standing 196 feet from the base of an office building in downtown Salt Lake City. The angle of elevation to the top of the building is 65˚. Find the height of the building. Assume you are 6 feet tall.



$$196 \cdot \tan 65 = \frac{h}{196} \cdot 196$$

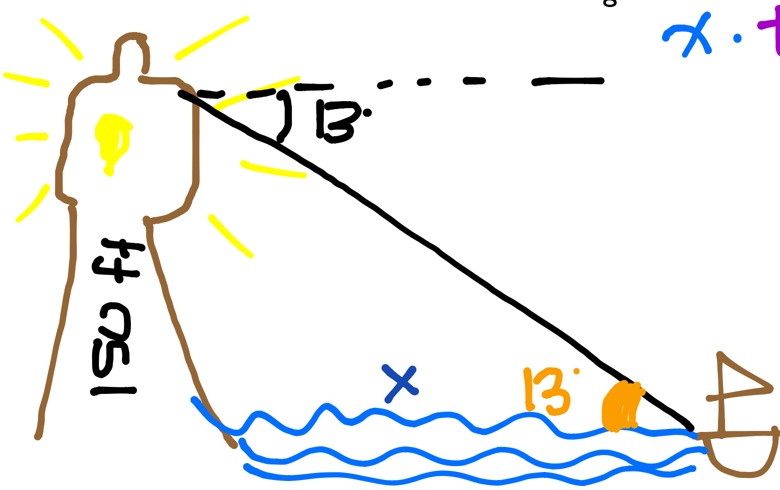
$$h = 196\tan 65$$

$$h = 420.323 \text{ ft}$$

$$+6 \text{ ft}$$

$$\boxed{\text{Height of the Building}: 426.323 \text{ ft}}$$

Example 7: The angle of depression from the top of a lighthouse 150 feet above the surface of the water to a boat is 13˚. How far is the boat from the lighthouse?



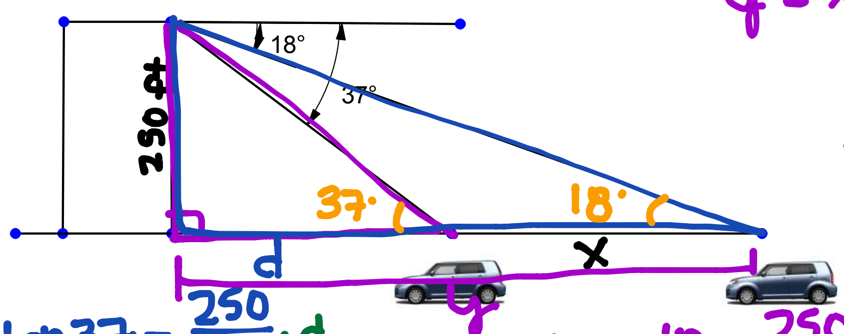
$$x \cdot \tan 13 = \frac{150}{x} \cdot x$$

$$\frac{x\cancel{\tan 13}}{\cancel{\tan 13}} = \frac{150}{\tan 13}$$

$$x = \frac{150}{\tan 13}$$

$$\boxed{x \approx 649.721 \text{ ft}}$$

Example 8: From the top of a building 250 feet high, a man observes a car moving toward him. If the angle of depression to the car changes from 18˚ to 37˚, how far does the car travel while the man is observing?



$$y = x + d$$

$$x = y - d$$

$$x = 769.421 - 331.761$$

$$\boxed{x \approx 437.660 \text{ ft}}$$

$$d \cdot \tan 37 = \frac{250}{d} \cdot d$$

$$\frac{d\tan 37}{\tan 37} = \frac{250}{\tan 37}$$

$$d \approx 331.761 \text{ ft}$$

$$y \cdot \tan 18 = \frac{250}{y} \cdot y$$

$$y\tan 18 = 250$$

$$y = \frac{250}{\tan 18}$$

$$y \approx 769.421 \text{ ft}$$