NOTES: SECONDARY 2 HONORS

UNIT 9: Exploring Trigonometric Ratios

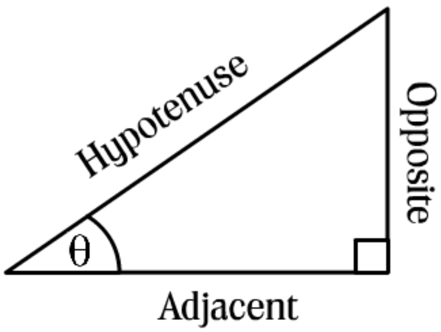
Essential Questions:

1. How are the properties of similar triangles used to create trigonometric ratios?

2. What are the relationships between the sides and the angles of right triangles?

3. On what variable inputs are the ratios in trigonometry dependent?

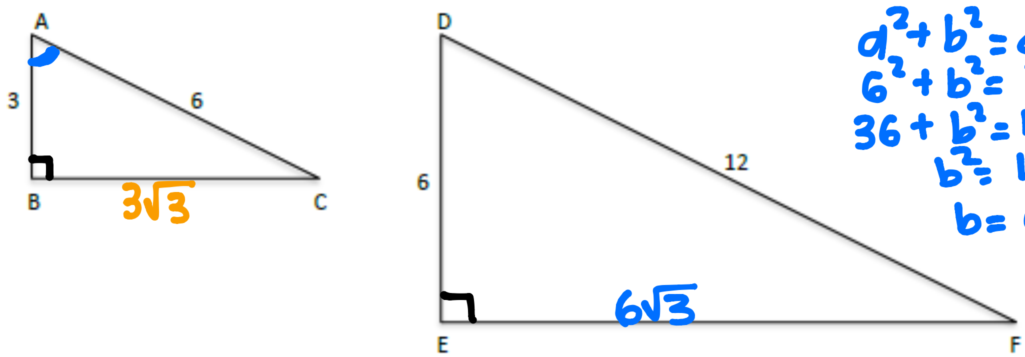
4. What is the relationship between the sine and cosine ratios for the two acute angles in a right triangle?



Theta (θ) is a Greek letter commonly used to represent an unknown angle measure.

Introduction: List the given ratios for the similar right triangles below.

$a^2+b^2=c^2$
$3^2+b^2=6^2$
$9+b^2=36$
$b^2=27$
$b=3\sqrt{3}$



$a^2+b^2=c^2$
$6^2+b^2=12^2$
$36+b^2=144$
$b^2=108$
$b=6\sqrt{3}$

List the ratios for ΔABC using angle A as the angle of reference.

$\dfrac{BC}{AC}$ $\dfrac{\textit{opposite side}}{\textit{hypotenuse}} = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2}$

$\dfrac{AB}{AC}$ $\dfrac{\textit{adjacent side}}{\textit{hypotenuse}} = \dfrac{3}{6} = \dfrac{1}{2}$

$\dfrac{BC}{AB}$ $\dfrac{\textit{opposite side}}{\textit{adjacent side}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

List the ratios for ΔDEF using angle D as the angle of reference.

$\dfrac{EF}{DF}$ $\dfrac{\textit{opposite side}}{\textit{hypotenuse}} = \dfrac{6\sqrt{3}}{12} = \dfrac{\sqrt{3}}{2}$

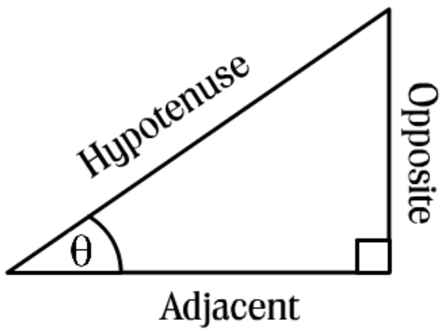
$\dfrac{DE}{DF}$ $\dfrac{\textit{adjacent side}}{\textit{hypotenuse}} = \dfrac{6}{12} = \dfrac{1}{2}$

$\dfrac{EF}{DE}$ $\dfrac{\textit{opposite side}}{\textit{adjacent side}} = \dfrac{6\sqrt{3}}{6} = \sqrt{3}$

What do you notice about the ratios for the two similar triangles?

They are the same!

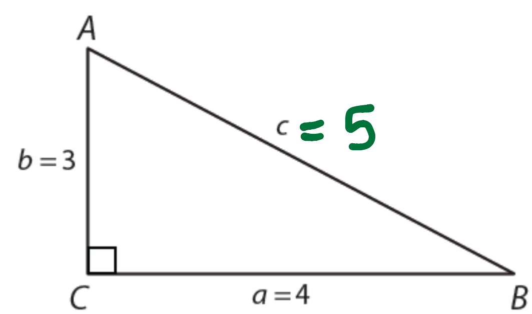
Trigonometry: The study of triangles and the relationships between their sides and the angles between these sides.



SOH CAH TOA

sine of ϑ = sin ϑ = $\dfrac{opposite}{hypotenuse}$	**cosecant ϑ = csc ϑ =** $\dfrac{1}{\sin\theta} = \dfrac{hypotenuse}{opposite}$
cosine ϑ = cos ϑ = $\dfrac{adjacent}{hypotenuse}$	**secant ϑ = sec ϑ =** $\dfrac{1}{\cos\theta} = \dfrac{hypotenuse}{adjacent}$
tangent ϑ = tan ϑ = $\dfrac{opposite}{adjacent}$	**cotangent ϑ = cot ϑ =** $\dfrac{1}{\tan\theta} = \dfrac{adjacent}{opposite}$

Example 1: Find the sine, cosine, tangent, cosecant, secant, and cotangent ratios for ∠A and ∠B.



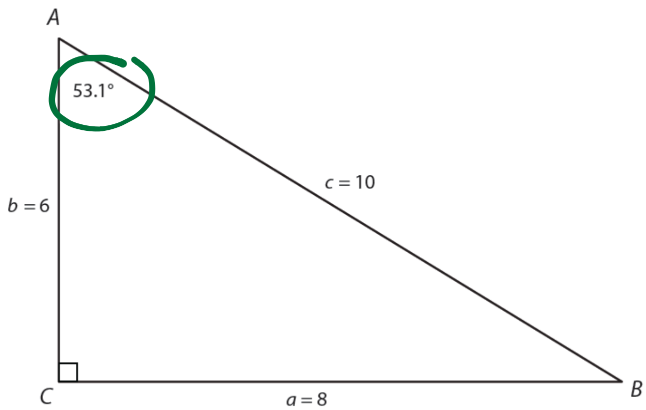
∠A		∠B	
sin A = $\frac{opp}{hyp} = \frac{4}{5}$	csc A = $\frac{hyp}{opp} = \frac{5}{4}$	sin B = $\frac{3}{5}$	csc B = $\frac{5}{3}$
cos A = $\frac{adj}{hyp} = \frac{3}{5}$	sec A = $\frac{hyp}{adj} = \frac{5}{3}$	cos B = $\frac{4}{5}$	sec B = $\frac{5}{4}$
tan A = $\frac{opp}{adj} = \frac{4}{3}$	cot A = $\frac{adj}{opp} = \frac{3}{4}$	tan B = $\frac{3}{4}$	cot B = $\frac{4}{3}$

Also, in the example above, notice that:

$\sin\theta = \frac{1}{\csc\theta}$ $\qquad$ $\cos\theta = \frac{1}{\sec\theta}$ $\qquad$ $\tan\theta = \frac{1}{\cot\theta}$

Example 2: Given the triangle below, set up the three trigonometric ratios of sine, cosine, and tangent for the reference angle given. Convert the ratios to decimal equivalents. Round to the nearest thousandth. Compare these ratios to the trigonometric functions using your calculator.

*Make sure your calculator is in **degree** MODE



Trigonometric Ratio	Ratio as a decimal	Calculator result
sin 53.1 = $\frac{8}{10}$	0.8	0.8
cos 53.1 = $\frac{6}{10}$	0.6	0.6
tan 53.1 = $\frac{8}{6}$	1.333	1.332

Use the triangle above and the information in the table above to determine the following:

Trigonometric Ratio	Ratio as a decimal	Calculator result
csc 53.1 = $\frac{10}{8}$	1.25	1.25
sec 53.1 = $\frac{10}{6}$	1.667	1.666
cot 53.1 = $\frac{6}{8}$	0.75	0.75

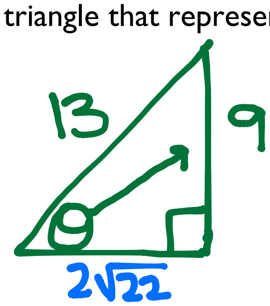
Example 3: Draw and label a right triangle that represents the relationship:

a. $\sin\theta = \frac{9}{13}$ $\frac{opp}{hyp}$

What is $\cos\theta$? $\boxed{\frac{2\sqrt{22}}{13}}$

$\tan\theta$? $\frac{9}{2\sqrt{22}} \cdot \frac{\sqrt{22}}{\sqrt{22}} = \frac{9\sqrt{22}}{2 \cdot 22}$

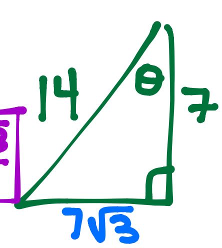
$= \boxed{\frac{9\sqrt{22}}{44}}$



b. $\cos\theta = \frac{7}{14}$ $\frac{adj}{hyp}$

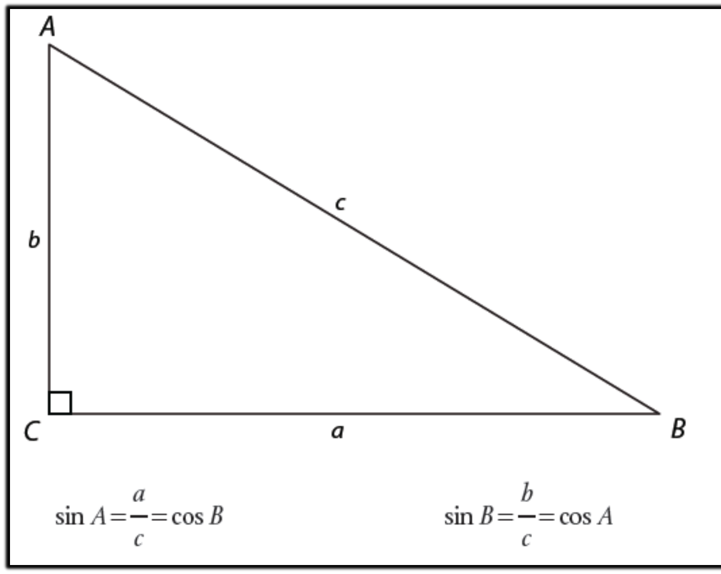
What is $\sin\theta$? $\frac{7\sqrt{3}}{14} = \boxed{\frac{\sqrt{3}}{2}}$

$\tan\theta$? $\frac{7\sqrt{3}}{7} = \boxed{\sqrt{3}}$



Cofunction Identities

- ➔ In the triangle at the right, notice that sin A = cos B and sin B = cos A



- ➔ Sine and cosine are called **cofunctions** because the value of one ratio for one angle is the same as the value of the other ratio for the other angle. This relationship between sine and cosine is referred to as a **cofunction identity**. Another way of writing the cofunction identity is:

$$\sin\theta = \cos(90° - \theta)$$
$$\cos\theta = \sin(90° - \theta)$$

- ➔ What are two other cofunction identities?

$\csc\theta = \sec(90° - \theta)$

$\sec\theta = \csc(90° - \theta)$

Example 4: Complete the tables below using the sine and cosine identities.

Angle	Sine	Cosine
10º	0.174	0.985
80º	0.985	0.174

Angle	Sine	Cosine
28º	0.470	0.883
62°	0.883	0.470

Example 5:

a. For which value of θ is sin θ = cos 64˚? $\theta = 26°$

b. For which value of θ is cos θ = sin 45˚? $\theta = 45°$

c. For which value of θ is csc θ = sec 21˚? $\theta = 69°$