NOTES: SECONDARY 2H
SIMPLIFYING TRIGONOMETRIC IDENTITIES

VOCAB:

- A **trigonometric identity** is a statement of equality that is true for ***all*** values of the variable for which both sides of the equation are defined.

Basic Trigonometric Identities

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad \cos\theta = \dfrac{1}{\sec\theta}$ $\qquad \tan\theta = \dfrac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

When ***simplifying*** trigonometric expressions, we manipulate the expression using identities until we get the most simplified answer. Your final answer should not have any fractions.

****A trick that is sometimes helpful is to replace all trig functions with sine and cosine.****

Example 1: Simplify the following trig expressions.

a. $\cot x \cdot \sin x$

$\dfrac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} = \boxed{\cos x}$

b. $\csc x \cdot \tan x$

$\dfrac{1}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{\cos x} = \dfrac{1}{\cos x} = \boxed{\sec x}$

c. $\dfrac{\cot\theta}{\csc\theta}$

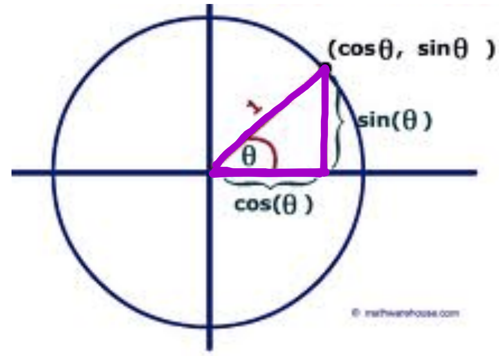
$\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \dfrac{\cos\theta}{\cancel{\sin\theta}} \cdot \dfrac{\cancel{\sin\theta}}{1} = \boxed{\cos\theta}$

d. $\dfrac{\sec x \cdot \csc x}{\tan x \cdot \cot x}$

$\dfrac{\frac{1}{\cos x} \cdot \frac{1}{\sin x}}{\frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}}} = \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}$

$= \boxed{\sec x \csc x}$

By the Pythagorean Theorem we know that $\sin^2 x + \cos^2 x = 1$



⇒ Divide every term by $\cos^2 x$ and simplify (no fractions!) your result:

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

⇒ This time, divide every term by $\sin^2 x$ and simplify your result:

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$

These relationships are called the **Pythagorean Identities**.

We can manipulate the Pythagorean Identities to find more equivalent statements.

I. Use the Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

1. Solve the above Pythagorean Identity for $\sin^2 x$

$$\sin^2 x + \cos^2 x = 1$$
$$-\cos^2 x \quad -\cos^2 x$$
$$\boxed{\sin^2 x = 1 - \cos^2 x}$$

2. Solve the above Pythagorean Identity for $\cos^2 x$

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

II. Use the Pythagorean Identity: $\tan^2 x + 1 = \sec^2 x$

3. Solve the above Pythagorean Identity for $\tan^2 x$

$$\boxed{\tan^2 x = \sec^2 x - 1}$$

4. Solve the above Pythagorean Identity for 1

$$\boxed{1 = \sec^2 x - \tan^2 x}$$

III. Use the Pythagorean Identity: $1 + \cot^2 x = \csc^2 x$

5. Solve the above Pythagorean Identity for 1

$$\boxed{1 = \csc^2 x - \cot^2 x}$$

6. Solve the above Pythagorean Identity for $\cot^2 x$

$$\boxed{\cot^2 x = \csc^2 x - 1}$$

Example 2: Simplify the following trigonometric expressions.

a. $\frac{1+\tan^2 x}{\sin^2 x + \cos^2 x}$

$$= \frac{\sec^2 x}{1} = \boxed{\sec^2 x}$$

b. $\cos^2 x + \tan^2 x + \sin^2 x$

$$= 1$$

$$1 + \tan^2 x = \boxed{\sec^2 x}$$

Sometimes when we simplify a trigonometric identity, we need to either factor or distribute to expand.

$$9x^2 - 16 = (3x-4)(3x+4)$$
$$1 - \cos^2 x = (1-\cos x)(1+\cos x)$$

Example 3: Simplify the following trigonometric expressions.

a. $\sin^3 x + \sin x \cos^2 x$

$\sin x(\underbrace{\sin^2 x + \cos^2 x}_{=1})$

$\sin x(1) = \boxed{\sin x}$

b. $1 - 2\sin x + \sin^2 x$

$u = \sin x$

$1 - 2u + u^2 = u^2 - 2u + 1$

$(u-1)(u-1) = (u-1)^2$

$= \boxed{(\sin x - 1)^2}$

c. $\cos^4 x - 2\cos^2 x + 1$

$u = \cos^2 x$

$u^2 - 2u + 1 = (u-1)^2$

$(\cos^2 x - 1)^2$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \cos^2 x - 1 = 0$

$-\sin^2 x \qquad -\sin^2 x$

$\cos^2 x - 1 = -\sin^2 x$

$(-\sin^2 x)^2$

$= \boxed{\sin^4 x}$

d. $\frac{(\sec x + 1)(\sec x - 1)}{\sin^2 x} = \frac{\sec^2 x - \cancel{\sec x + \sec x} - 1}{\sin^2 x}$

$\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x}$

$\frac{\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \frac{\cancel{\sin^2 x}}{\cos^2 x} \cdot \frac{1}{\cancel{\sin^2 x}} = \frac{1}{\cos^2 x}$

$= \boxed{\sec^2 x}$

Cofunction Identities		**Odd-Even Identities**	
		$f(-x) = -f(x)$ Odd Functions	$f(-x) = f(x)$ Even Functions
$\sin\theta = \cos(90° - \theta)$	$\cos\theta = \sin(90° - \theta)$	$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$
$\tan\theta = \cot(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$	$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$
$\csc\theta = \sec(90° - \theta)$	$\sec\theta = \csc(90° - \theta)$	$\tan(-x) = -\tan x$	
		$\cot(-x) = -\cot x$	

Example 4: Simplify the following trigonometric expressions.

a. $\cos x \sin(-x) \sec^2 x$

$\cancel{\cos x}(-\sin x)\frac{1}{\cos^{\cancel{2}} x}$

$= \frac{-\sin x}{\cos x} = \boxed{-\tan x}$

b. $\cos(-x)\csc(-x)\tan x$

$\cos x(-\csc(x))\left(\frac{\sin x}{\cos x}\right)$

$\cancel{\cos x}\left(\frac{-1}{\cancel{\sin x}}\right)\left(\frac{\cancel{\sin x}}{\cancel{\cos x}}\right) = \boxed{-1}$

c. $-\frac{\cos(90° - x)}{\sin x}$

$-\frac{\sin x}{\sin x} = \boxed{-1}$

d. $\cot(90° - x)\frac{1}{\sec(-x)}\sin(90° - x)$

$\tan x(\cos(-x))(\cos x)$

$\tan x(\cos x)(\cos x)$

$\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x} \cdot \cos x$

$= \boxed{\sin x \cdot \cos x}$