**NOTES: SECONDARY 2H**
Unit 9: Solving Trigonometric Equations

Solve the following for the unknown:

| | | |
|---|---|---|
| 1. $2x - 1 = 0$ | 2. $4x^2 - 3 = 0$ | 3. $9x^2 - 25 = 0$ |
| 4. $2x^2 + 5x - 12 = 0$ | 5. $\sin 30° = x$ | 6. $\sin 150° = x$ |

Question:

- How many answers do you have to the problem $\sin\theta = \frac{1}{2}$?

infinite

- How many answers would you have if I restricted your solutions to $0° \le \theta \le 360°$?

TWO solutions (30° & 150°)

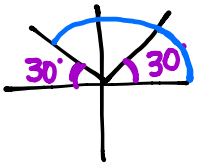
When solving trigonometric equations, there a couple of steps we can follow.

**STEP #1:** Solve for the trigonometric equation. (So the only term on one side of the equal sign is a trigonometric function and everything else is on the other side.)
**STEP #2:** Determine what two quadrants your solution will fall in. (Between $0° \le \theta \le 360°$, there will always be two quadrants.)
**STEP #3:** Solve for your reference angle using the table of the 9 basic trig functions.
**STEP #4:** Using the reference angle in #3 and the quadrants in #2, write your final solution. Remember, your answer should be a positive angle between $0°$ and $360°$.

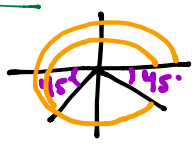
S | A
T | C

Example 1: Find the values of $x$ if $0° \le x \le 360°$.

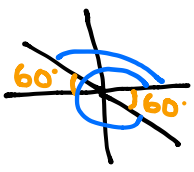
a. $2\sin x - 1 = 0$



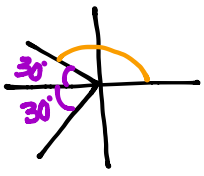

b. $\sin x + \sqrt{2} = -\sin x$




c. $2\tan x + 2\sqrt{3} = 0$




d. $2\cos x + \sqrt{3} = 0$



Sometimes, we need to factor before we can solve.

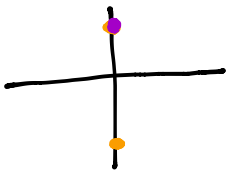
Example 2: Find the values of $\theta$ if $0° \le \theta \le 360°$.

a. $\cos\theta - \cos\theta \sin\theta = 0$

$\cos\theta(1-\sin\theta)=0$

$\cos\theta=0$ $\quad$ $1-\sin\theta=0$, $\sin\theta=1$, $\theta=90°$

$\theta = 90°, 270°$

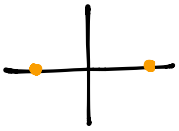


b. $\sin^2\theta - 2\sin\theta = 0$

$\sin\theta(\sin\theta-2)=0$

$\sin\theta=0$ $\quad$ $\sin\theta-2=0$, $\sin\theta=2$

$\theta=0°, 180°, 360°$



c. $2\sin^2\theta + \sin\theta = 1$

$2\sin^2\theta+\sin\theta-1=0$ $\quad$ $u=\sin\theta$

$2u^2+u-1=0$

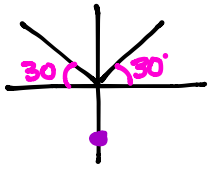
$(2u-1)(u+1)=0$

$+2u$ $-1u$

$2u-1=0$, $u=\frac{1}{2}$ $\quad$ $u+1=0$, $u=-1$

$\sin\theta=\frac{1}{2}$ $\quad$ $\sin\theta=-1$

$\theta=30, 150°, 270°$




d. $2\cos^2 x + \cos x - 1 = 0$ $\quad$ $u=\cos x$

$2u^2+u-1=0$

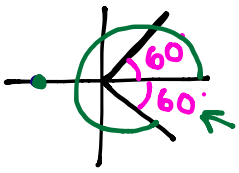
$(2u-1)(u+1)=0$

$u=\frac{1}{2}$ $\quad$ $u=-1$

$\cos x=\frac{1}{2}$ $\quad$ $\cos x=-1$

$x=60°, 300°, 180°$

$x=60°, 180°, 300°$




CHALLENGE PROBLEMS:

a. $\frac{\cos^3 x}{\sin x} = \cot x$

b. $2\sin^2 x - 3\cos x = 0$

$2(1-\cos^2 x)-3\cos x=0$ $\quad$ $u=\cos x$

$2-2\cos^2 x-3\cos x=0$

$-2\cos^2 x-3\cos x+2=0$

$+2u^2+3u+2=0(-1)$

$2u^2+3u-2=0$

$(2u-1)(u+2)=0$

$+4u$ $-1u$

$2u-1=0$, $u=\frac{1}{2}$ $\quad$ $u+2=0$, $u=-2$

$\cos x=\frac{1}{2}$ $\quad$ $\cos x=-2$

$x=60°, 300°$