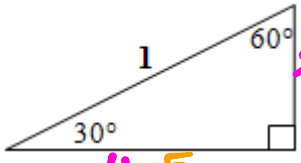
Special Right Triangles

Find the missing sides of the triangles.

30-60-90



$x = \frac{1}{2}$

$y = \frac{\sqrt{3}}{2}$

$\left(\frac{1}{2}\right)^2 + y^2 = (1)^2$

$\frac{1}{4} + y^2 = 1$

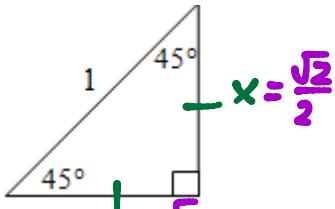
$y^2 = \frac{3}{4}$

$y = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{x}{1} = x$

$x = \frac{1}{2}$

45-45-90



$x = \frac{\sqrt{2}}{2}$

$x = \frac{\sqrt{2}}{2}$

$x^2 + x^2 = 1$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{2}$

Complete the table of values below. You will want to **MEMORIZE** these trig values.

	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Example 1: Answer the following without using a calculator.

a. $\sin 30°$ $\frac{1}{2}$

b. $\tan 45°$ 1

c. $\cos 30°$ $\frac{\sqrt{3}}{2}$

d. $\cos 60°$ $\frac{1}{2}$

e. $\cos 45°$ $\frac{\sqrt{2}}{2}$

f. $\cot 30°$ $\sqrt{3}$

$\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3}$

g. $\csc 45°$ $\sqrt{2}$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

h. $\sin 60°$ $\frac{\sqrt{3}}{2}$

Example 2: Find θ without using a calculator.

a. $\cos\theta = \frac{1}{2}$ $\theta = 60°$

b. $\sin\theta = \frac{\sqrt{2}}{2}$ $\theta = 45°$

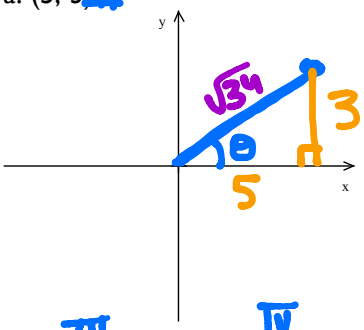
c. $\tan\theta = \sqrt{3}$ $\theta = 60°$

d. $\cos\theta = \frac{\sqrt{3}}{2}$ $\theta = 30°$

Evaluating Trig Functions Determined by a Point

Let θ be the acute angle in standard position whose terminal side contains the given point. Find the six trigonometric functions of θ.

a. (5, 3)



III I

$3^2+5^2=c^2$

$9+25=c^2$

$34=c^2$

$c=\sqrt{34}$

III IV

$\sin\theta=\frac{3}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}=\frac{3\sqrt{34}}{34}$

$\cos\theta=\frac{5}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}=\frac{5\sqrt{34}}{34}$

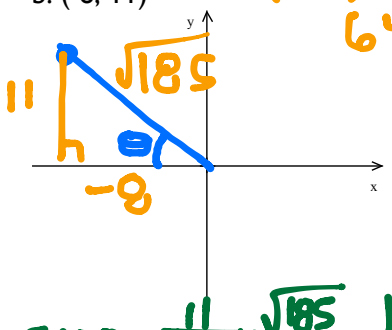
$\tan\theta=\frac{3}{5}$

$\csc\theta=\frac{\sqrt{34}}{3}$

$\sec\theta=\frac{\sqrt{34}}{5}$

$\cot\theta=\frac{5}{3}$

b. (-8, 11)



$(-8)^2+(11)^2=c^2$

$64+121=c^2$

$c^2=185$

$c=\sqrt{185}$

$\sin\theta=\frac{11}{\sqrt{185}}\cdot\frac{\sqrt{185}}{\sqrt{185}}=\frac{11\sqrt{185}}{185}$

$\cos\theta=\frac{-8}{\sqrt{185}}\cdot\frac{\sqrt{185}}{\sqrt{185}}=\frac{-8\sqrt{185}}{185}$

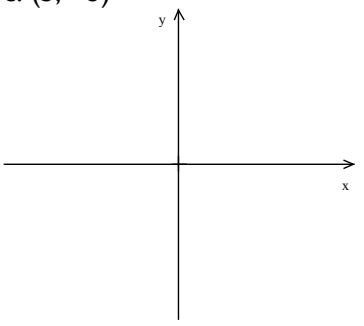
$\tan\theta=\frac{11}{-8}$

$\csc\theta=\frac{\sqrt{185}}{11}$

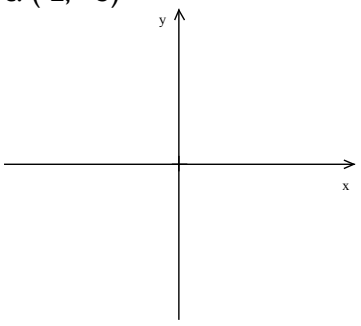
$\sec\theta=\frac{\sqrt{185}}{-8}$

$\cot\theta=\frac{-8}{11}$

c. (5, −3)



c. (-2, −5)



Quadrantal Angles: Quadrantal Angles are angles that fall on an axis. To find the trig functions of these angles, we use the ordered pair of the point $(x,y)=(\cos\theta,\sin\theta)$ and $\tan x=\frac{\sin x}{\cos x}=\frac{y}{x}$ This only works on the unit circle (when the radius is 1 unit).

a. $\sin 180°$ 0

b. $\cos 270°$ 0

c. $\tan 90°$ undefined

$\frac{1}{0}$

d. $\tan 360°$ 0

$\frac{0}{1}$