NOTES: SECONDARY 2H
VERIFYING TRIGONOMETRIC IDENTITIES

Last class we talked about ***simplifying*** trigonometric identities. Today, we are going to talk about ***verifying*** trigonometric identities. When proving an identity, the end result of what you want is on the other side of the equal sign. Your work is the steps shown how to get from one side of the equation to the other side of the equation. Typically, **you want to start with the more complex side** only and match it to the other side. There may be some problems where you will work from both sides to get to a common expression.

Example 1: Prove the identity.

a. $\tan x + \cot x = \sec x \csc x$

$\tan x + \cot x = \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}$ LCD: $\cos x \sin x$

$= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ $=1$

$\frac{1}{\sin x \cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x} = \csc x \sec x$ ✓

b. $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 2\cot x \csc x$ LCD: $(\sec x - 1)(\sec x + 1)$

$\frac{\sec x + 1}{(\sec x + 1)(\sec x - 1)} + \frac{\sec x - 1}{(\sec x + 1)(\sec x - 1)}$

$= \frac{\sec x \cancel{+1} + \sec x \cancel{-1}}{\sec^2 x - 1} = \frac{2\sec x}{\tan^2 x} = 2\sec x\left(\frac{1}{\tan^2 x}\right)$

$= 2\left(\frac{1}{\cancel{\cos x}}\right)\left(\frac{\cos^{\cancel{2}} x}{\sin^2 x}\right)$

$2\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\sin x}\right)$

$= 2\cot x \csc x$ ✓

c. $\frac{\tan y}{\sin y} = \sec y$

$\frac{\tan y}{\sin y} = \frac{\frac{\sin y}{\cos y}}{\sin y} = \frac{\cancel{\sin y}}{\cos y}\cdot\frac{1}{\cancel{\sin y}}$

$= \frac{1}{\cos y} = \sec y$ ✓

d. $\frac{\sin^2 x \cot x}{\cos x} = \sin x$

$\frac{\sin^{2} x\left(\frac{\cos x}{\cancel{\sin x}}\right)}{\cos x} = \frac{\sin x \cancel{\cos x}}{\cancel{\cos x}}$

$= \sin x$ ✓

e. $\frac{\cot^2 x}{(1+\csc x)} = (\cot x)(\sec x - \tan x)$

$\frac{\cot^2 x}{1+\csc x} = \frac{\csc^2 x - 1}{1+\csc x} = \frac{(\csc x - 1)\cancel{(\csc x + 1)}}{\cancel{1+\csc x}}$

$= \csc x - 1$

$(\cot x)(\sec x - \tan x) = \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$

$= \frac{1}{\sin x} - 1 = \csc x - 1$ ✓

f. $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2\sec x$ LCD: $(1+\sin x)(\cos x)$

$\frac{\cos^2 x + (1+\sin x)(1+\sin x)}{\cos x(1+\sin x)}$

$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)(\cos x)}$

$= \frac{1+1+2\sin x}{(1+\sin x)(\cos x)} = \frac{2+2\sin x}{(1+\sin x)(\cos x)}$

$= \frac{2\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(\cos x)} = \frac{2}{\cos x} = 2\sec x$ ✓

g. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

$\frac{1}{\cos^2 x}\cdot\frac{\sin^2 x}{\sin^2 x} + \frac{1}{\sin^2 x}\cdot\frac{\cos^2 x}{\cos^2 x}$ LCD: $\cos^2 x \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$ $=1$

$\frac{1}{\cos^2 x \sin^2 x} = \sec^2 x + \csc^2 x$ ✓

h. $(1 - \sin x)(1 + \sin x)\csc^2 x + 1 = \csc^2 x$

$(1-\sin^2 x)(\csc^2 x) + 1$

$= (\cos^2 x)\left(\frac{1}{\sin^2 x}\right) + 1$

$= \frac{\cos^2 x}{\sin^2 x} + 1$

$= \cot^2 x + 1 = \csc^2 x$ ✓

i. $1 - \cos x \tan x \sin x = \cos^2 x$

$1 - \frac{\cancel{\cos x}}{1}\cdot\frac{\sin x}{\cancel{\cos x}}\cdot\frac{\sin x}{1}$

$1 - \sin^2 x = \cos^2 x$ ✓

j. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$(1 - \sin^2 x) - \sin^2 x$

$= 1 - 2\sin^2 x$ ✓

k. $(1 - \cos^2 x)(1 + \cot^2 x) = 1$

$(\sin^2 x)(\csc^2 x)$

$= (\sin^2 x)\left(\frac{1}{\sin^2 x}\right) = \frac{\sin^2 x}{\sin^2 x} = 1$ ✓

l. $\sin\theta\,(\csc\theta - \sin\theta) = \cos^2\theta$

$\sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$

$= 1 - \sin^2\theta = \cos^2\theta$ ✓

m. $\cos x - \cos x \sin^2 x = \cos^3 x$

$\cos x(1 - \sin^2 x)$

$= \cos x(\cos^2 x) = \cos^3 x$ ✓

n. $\tan x + \cot x = \sec x \csc x$

$\frac{\sin x\cdot\sin x}{\sin x\cdot\cos x} + \frac{\cos x\cdot\cos x}{\sin x\cdot\cos x}$ LCD: $\sin x\cos x$

$\frac{\sin^2 x}{\sin x\cos x} + \frac{\cos^2 x}{\sin x\cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ $=1$

$\frac{1}{\sin x\cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x} = \csc x\sec x$ ✓

o. $\underbrace{\sin(90° - x)\csc x}_{} = \cot x$

$\cos(x)\csc x$

$\frac{\cos x}{1}\left(\frac{1}{\sin x}\right) = \frac{\cos x}{\sin x} = \cot x$ ✓

p. $\underbrace{\tan(-x)\cos x}_{} = -\sin x$

$-\tan(x)\cos x$

$= \frac{-\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x} = -\sin x$ ✓

q. $(\sec x - \tan x) = \underbrace{\frac{1-\sin x}{\cos x}}_{}$

$\frac{1-\sin x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$= \sec x - \tan x$ ✓

r. $\cos x \cot x = \underbrace{\frac{1-\sin^2 x}{\sin x}}_{}$

$= \frac{\cos^2 x}{\sin x} = \cos x\left(\frac{\cos x}{\sin x}\right)$

$= \cos x \cot x$ ✓

s. $1 + \sec x = \underbrace{\frac{1+\cos x}{\cos x}}_{}$

$\frac{1+\cos x}{\cos x} = \frac{1}{\cos x} + \frac{\cos x}{\cos x}$

$= \sec x + 1$

$= 1 + \sec x$ ✓

t. $\underbrace{(\cot x + 1)^2}_{} = \csc^2 x + \underline{2\cot x}$

$(\cot x + 1)(\cot x + 1)$

$\cot^2 x + \underline{2\cot x} + 1$

$= \csc^2 x + 2\cot x$ ✓