NOTES: SECONDARY 2H
SIMPLIFYING TRIGONOMETRIC IDENTITIES

VOCAB:

- A **trigonometric identity** is a statement of equality that is true for ***all*** values of the variable for which both sides of the equation are defined.

Basic Trigonometric Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

When ***simplifying*** trigonometric expressions, we manipulate the expression using identities until we get the most simplified answer. Your final answer should not have any fractions.

****A trick that is sometimes helpful is to replace all trig functions with sine and cosine.****

Example 1: Simplify the following trig expressions.

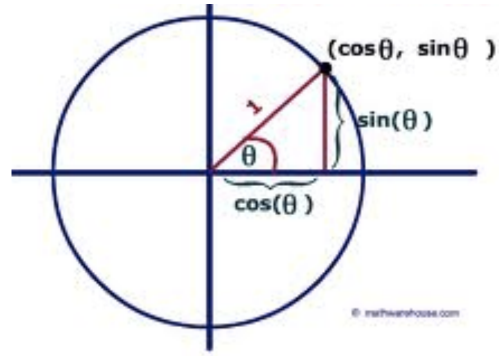
a. $\cot x \cdot \sin x$

b. $\csc x \cdot \tan x$

c. $\frac{\cot\theta}{csc\theta}$

d. $\frac{\sec x \cdot \csc x}{\tan x \cdot \cot x}$

By the Pythagorean Theorem we know that $\sin^2 x + \cos^2 x = 1$



⇒ Divide every term by $\cos^2 x$ and simplify (no fractions!) your result:

⇒ This time, divide every term by $\sin^2 x$ and simplify your result:

These relationships are called the **Pythagorean Identities**.

We can manipulate the Pythagorean Identities to find more equivalent statements.

I. Use the Pythagorean Identity: $\mathbf{\sin^2 x + \cos^2 x = 1}$

1. Solve the above Pythagorean Identity for $\sin^2 x$

2. Solve the above Pythagorean Identity for $\cos^2 x$

II. Use the Pythagorean Identity: $\mathbf{\tan^2 x + 1 = \sec^2 x}$

3. Solve the above Pythagorean Identity for $\tan^2 x$

4. Solve the above Pythagorean Identity for 1

III. Use the Pythagorean Identity: $\mathbf{1 + \cot^2 x = \csc^2 x}$

5. Solve the above Pythagorean Identity for 1

6. Solve the above Pythagorean Identity for $\cot^2 x$

Example 2: Simplify the following trigonometric expressions.

a. $\frac{1+\tan^2 x}{\sin^2 x+\cos^2 x}$

b. $\cos^2 x + \tan^2 x + \sin^2 x$

Sometimes when we simplify a trigonometric identity, we need to either factor or distribute to expand.

Example 3: Simplify the following trigonometric expressions.

a. $\sin^3 x + \sin x \cos^2 x$

b. $1 - 2\sin x + \sin^2 x$

c. $\cos^4 x - 2\cos^2 x + 1$

d. $\frac{(\sec x+1)(\sec x-1)}{\sin^2 x}$

Cofunction Identities		**Odd-Even Identities**	
		Odd Functions	Even Functions
$\sin\theta = \cos(90° - \theta)$	$\cos\theta = \sin(90° - \theta)$	$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$
$\tan\theta = \cot(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$	$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$
$\csc\theta = \sec(90° - \theta)$	$\sec\theta = \csc(90° - \theta)$	$\tan(-x) = -\tan x$	
		$\cot(-x) = -\cot x$	

Example 4: Simplify the following trigonometric expressions.

a. $\cos x \sin(-x) \sec^2 x$

b. $\cos(-x)\csc(-x)\tan x$

c. $-\frac{\cos(90°-x)}{\sin x}$

d. $\cot(90° - x)\frac{1}{\sec(-x)}\sin(90° - x)$