

NOTES: SECONDARY 2H
Unit 9: Sum/Differences Identities

Sum and Difference Identities

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Example 1: Find the exact value of $\sin 15^\circ$ without using a calculator.

STEP #1: Rewrite the angle as a sum or difference statement using the angles we know (30° , 45° , 60°).

STEP #2: Use the identities above to rewrite $\sin 15^\circ$.

STEP #3: Evaluate the above identities using the values on the table (you should have memorized).

Example 2: Find the exact value of the following using the sum and difference identities.

a. $\cos 75^\circ$

b. $\tan(-15^\circ)$

c. $\sin 105^\circ$

d. $\cos 120^\circ$

We can also include all the obtuse angles we have found. (30° , 45° , 60° , 90° , 120° , 135° , 150° , 180° , 210° , 225° , 240° , 270° , 300° , 315° , 330° , 360°)

Example 3: Find the exact value of the following using the sum and difference identities.

a. $\sin 165^\circ$

b. $\cos 255^\circ$

c. $\tan 195^\circ$

d. $\cos 285^\circ$

Example 4: Using the trig identities, write the following expressions as the sine or cosine of an angle.

a. $\sin 22^\circ \cos 13^\circ + \cos 22^\circ \sin 13^\circ$

b. $\sin x^\circ \sin 2x^\circ - \cos x^\circ \cos 2x^\circ$

c. $\cos 60^\circ \cos 45^\circ + \sin 45^\circ \sin 60^\circ$